

# Application of Multiobjective Design Exploration to SOLAR-C Orbit Design

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**Multiobjective design exploration (MODE) framework is applied to the trajectory design problem for SOLAR-C Plan A SEP option. Present approach revealed some important design knowledge such as tradeoff information among maximization of final mass, maximization of final velocity, and maximization of minimum distance from the Sun. Present study also shows possibility of simultaneous design of trajectory and spacecraft.**

## SOLAR-C 軌道設計への多目的設計探査の適用

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SOLAR-C Plan A SEP オプションの軌道設計問題に対して、多目的設計探査を行った。その結果、終端質量最大化、終端速度最大化、最小動径半径の最大化の間のトレードオフ関係などの軌道設計上有益な知見が得られた。また、軌道・機体同時設計の可能性が示された。

### Nomenclature

|                |   |  |
|----------------|---|--|
| $\mathbf{a}_i$ | = | acceleration vector at segment point $i$                                 |
| $a_{i,x}$      | = | $x$ component of the acceleration vector $\mathbf{a}_i$                  |
| $a_{i,y}$      | = | $y$ component of the acceleration vector $\mathbf{a}_i$                  |
| $a_{i,z}$      | = | $z$ component of the acceleration vector $\mathbf{a}_i$                  |
| $A_z$          | = | azimuth angle at the first Earth swing-by (deg.)                         |
| $E_l$          | = | elevation angle at the first Earth swing-by (deg.)                       |
| $F$            | = | actual thrust available for maneuver (mN)                                |
| $F_{max}$      | = | maximum thrust of solar electric propulsion (mN)                         |
| $i$            | = | segment point number ( $i=1,14$ )  |
| $I_{sp}$       | = | specific impulse of solar electric propulsion (s)                        |
| $k_{op}$       | = | factor for the operation rate of the ion engine                          |
| $M$            | = | final mass of the spacecraft (kg)  |
| $r$            | = | distance from the Sun to the spacecraft (AU)                             |
| $r_E$          | = | distance from the Sun to the Earth (AU)                                  |
| $r_{min}$      | = | minimum distance from the Sun to the spacecraft (AU)                     |
| $V$            | = | velocity relative to the Earth at the first Earth swing-by (km/s)        |
| $x$            | = | a coordinate in the ecliptic plane (AU, see Fig. 3)                      |
| $y$            | = | a coordinate in the ecliptic plane (AU, see Fig. 3)                      |
| $z$            | = | coordinate in the direction normal to the ecliptic plane (AU, see Fig.3) |

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## I. Introduction

Members in the solar physics community have been studying the next Japanese solar observation mission coded as “SOLAR-C” whose launch is targeted on 2018. Currently, two possible plans are under discussion for the SOLAR-C mission. The first plan, which is called “Plan A”, aims at out-of-ecliptic magnetic/X-ray and helioseismic observations of the polar and the equatorial regions to investigate properties of polar region, meridional flow and magnetic structure inside the Sun down to bottom of the convection zone. To achieve this mission, observation from a high latitude point of the Sun is required where the target latitude is tentatively specified as 40 degrees (Figs.1 and 2). To observe the Sun from the high latitude point, the space observatory (the spacecraft) must be on the orbit largely inclined from the ecliptic plane. Some trajectory plans have been proposed and studied to inject the spacecraft into the target orbit<sup>1,2,3</sup>. One of them is solar electric propulsion option (SEP option)<sup>3</sup>. The SEP option adopts electric propulsion delta-v earth gravity assist (ED-VEGA) strategy, which bases on solar electric propulsion combined with Earth swing-by. Figure 3 presents trajectory of the current plan (it is called “nominal design” in this paper). As shown in this figure, the spacecraft orbits the Sun several times keeping the distance from the Sun at about 1(AU) to incline its trajectory with solar electric propulsion and Earth swing-by.

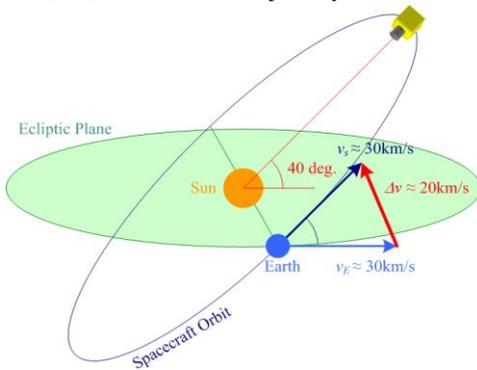


Fig.1 Mission of SOLAR-C Plan A

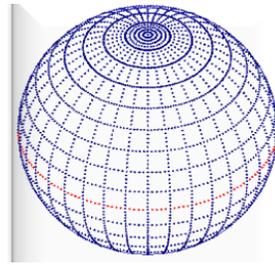


Fig.2 View of the Sun from 40 deg. Latitude.

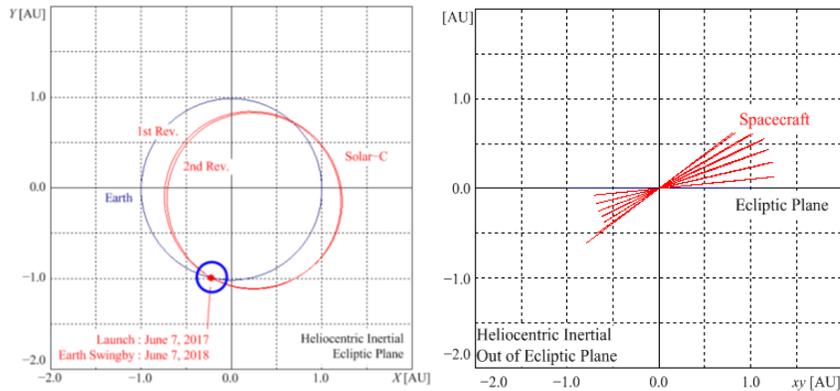


Fig.3 Trajectory of current design for SOLAR-C Plan A SEP option.

Design of trajectory for SEP option, however, is not an easy task. The first reason is that the number of design parameters is very large. If the thrust control profile is parameterized with acceleration vectors defined at 14 points as shown in Fig. 4, number of design parameters for each cycle is 42(=3x14). This results in 210 design parameters for five cycles. In addition, launch direction, swing-by conditions, and spacecraft design parameters (such as specific pulse and maximum thrust) should be optimized. The second reason is that multiple design objectives should be considered simultaneously. Traditionally, thrust control profile is optimized with the sequential quadratic programming (SQP) method so that the final mass of the spacecraft is maximized. However, maximization of velocity relative to the Earth should be considered at the same time. To reduce required weight for thermal protection system of the spacecraft, minimum distance from the Sun should be also maximized. Thus trajectory design problem for SEP option is a multiobjective design optimization problem.

While a single objective design optimization problem may have a unique optimal solution, a multiobjective design optimization problem has a set of compromised solutions, largely known as Pareto-optimal solutions or non-

dominated solutions. Each of these solutions is optimal in the sense that no other solutions in the search space are superior to it when all objectives are considered (Fig. 5).

Multiobjective design exploration<sup>4</sup> (MODE) is a framework to extract essential knowledge of a multiobjective design optimization problem, such as tradeoff information between contradicting objectives and the effect of each design parameter on the objectives. In the framework of MODE, non-dominated solutions are obtained by multiobjective optimization using, for example, a multiobjective evolutionary algorithm (see reference 5 for example), and important design knowledge is then extracted by analyzing objective function and design parameter values of the obtained non-dominated solutions using data mining approaches such as the self-organizing map and scattered plot matrix. Recently, MODE framework has been applied to a wide variety of design optimization problems including multidisciplinary design of a regional-jet wing<sup>6</sup>, aerodynamic design of a flapping airfoil<sup>7</sup>, and aerodynamic design of a turbine blade for a rocket engine<sup>8</sup>.

Objective of the present study is to show feasibility of MODE approach to trajectory design of a spacecraft with solar electric propulsion. MODE is applied to design of first cycle (from launch to the first Earth swing-by) for the Plan A SEP option for SOLAR-C mission.

## II. Formulation of Design Optimization Problem

Here, first cycle of the trajectory for Plan A SEP option is designed. The launcher is assumed to be the Japanese H2A heavy launch vehicle equipped with a solid motor upper stage. The initial mass of the spacecraft is assumed to be 1200(kg). The launch date is selected so as to take advantage of the not negligible tilt (7.25(deg.)) of the solar equatorial plane from the ecliptic plane. The design objectives are

- (1) maximization of the final mass (mass at the first Earth swing-by)
- (2) maximization of the final velocity relative to the Earth (velocity at the first Earth swing-by)
- (3) maximization of minimum distance from the Sun to the spacecraft.

The first design objective corresponds to minimization of fuel consumption during the first cycle. The second design objective corresponds to maximization of inclination angle (the velocity relative to the Earth is proportional to the inclination angle if the relative velocity can be turned to the direction normal to the trajectory plane by Earth swing-by). The third design objective corresponds to minimization of weight required for thermal protection system of the spacecraft (i.e., maximization of payload weight). The design constraints are

- (1) eccentricity should be less than 0.3
- (2) the product of specific impulse and the maximum thrust should be constant (150mN).

The design parameters are

- (1) azimuth angle of launch
- (2) elevation angle of launch
- (3) final velocity relative to the Earth
- (4) specific impulse of solar electric propulsion

and the thrust control profile during the first cycle. The thrust control profile is parameterized with acceleration vectors defined at 14 points as shown in Fig. 4. As a result, total number of design parameters turns out to be 46 (4+3x14). Please note that the final velocity relative to the Earth is a design parameter as well as a design objective to be maximized. Design space for the azimuth angle, elevation angle, final velocity, and specific impulse is

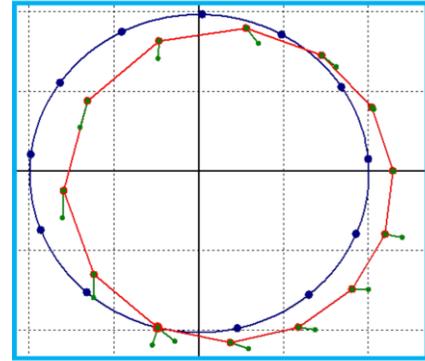


Fig.4 parameterization of the thrust control profile.

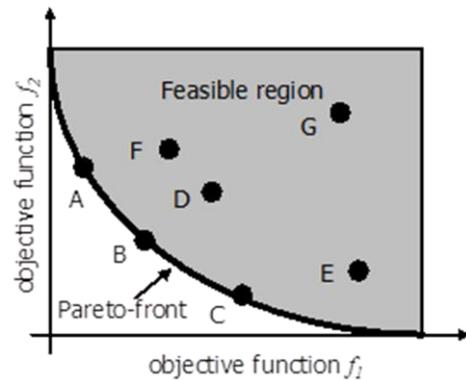


Fig.5 The concept of Pareto-optimality.

This is an example of multiobjective design optimization problems, which minimizes two conflicting objectives  $f_1$  and  $f_2$ . Gray-colored area is feasible region where solutions exist. This problem has innumerable non-dominated solutions such as solutions A, B, and C on the edge of the feasible region (Pareto-front). These solutions are optimal in the sense that there is no better solution in both objectives. One cannot say which is better among these non-dominated solutions because improvement in one objective degrades another.

shown in Table 1. Here, the actual thrust available for the maneuver is constrained as

$$F \leq \begin{cases} k_{op} F_{\max} & \text{if } r \leq r_E \\ k_{op} F_{\max} \left( \frac{r_E}{r} \right)^2 & \text{if } r > r_E \end{cases} \quad (1)$$

where  $k_{op} = 0.875$ .

**Table 1 Design space**

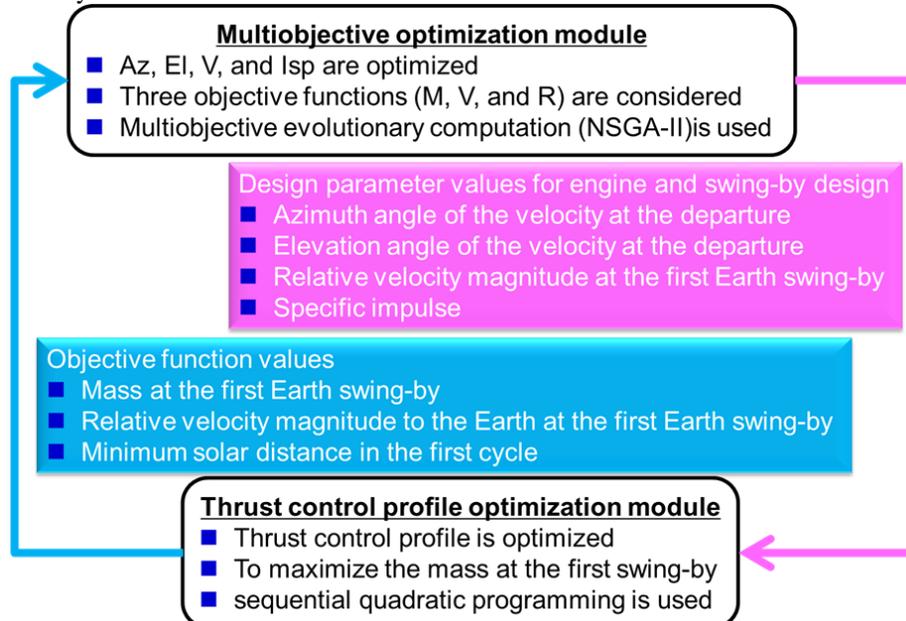
| Design parameter | Lower Limit | Nominal design | Upper limit |
|------------------|-------------|----------------|-------------|
| $A_z$            | -118.12     | 98.12          | -78.12      |
| $E_l$            | 1.80        | 21.80          | 41.80       |
| $V$              | 9.2         | 10.2           | 11.2        |
| $I_{sp}$         | 2400        | 3000           | 3600        |

### III. Computational method

As described in section I., we already had SQP software that optimizes thrust control profile for maximization of final mass. Therefore we constructed a hybrid optimization method based on SQP and multiobjective evolutionary algorithm (MOEA) where SQP optimizes thrust control profile and MOEA optimizes the azimuth angle, elevation angle, final velocity, and specific impulse. MOEA is used because it has capability to find non-dominated solutions efficiently.

The flowchart of the current hybrid optimization method is shown in Fig. 6. First, the multiobjective optimization module based on MOEA gives azimuth angle, elevation angle, final velocity, and specific impulse values of a population of design candidates to the thrust control profile optimization module. The thrust control profile optimization module optimizes the thrust control profiles of the design candidates for final mass maximization under given boundary conditions such as departure/arrival time, departure/arrival velocity relative to the Earth, and initial mass of the spacecraft and then gives back resulting objective function values (final mass, final velocity, and minimum distance from the Sun) of the population of design candidates. Then, the multiobjective optimization module generates a population of new design candidates using genetic operators such as selection, crossover, and mutation. This loop continues until number of generations (loops) reaches to a specified value.

For multiobjective optimization, non-dominated sorting genetic algorithm II (NSGA-II)<sup>9</sup> is used. The island model strategy with four islands is adopted to enhance evolution of the designs. The population size of each island is thirty-two except for the first generation. Because less than 10% of the design candidates are feasible (thrust control profile optimization module fail to output objective function values for most of the design candidates), the population size of each island is set to 160 for the first generation. Eight designs in each island is randomly chosen and moved to the next island every five generations. The optimization loop is repeated until number of generations (loops) reaches to forty. In total, the number of design candidates evaluated here is 5,760 (=160x4+32x4x40). Computational time was about two weeks on a Core i7 computer. Note that input parameter values of NSGA-II remain same as the default values of the NSGA-II source codes distributed on the website of Kanpur Genetic Algorithms Laboratory<sup>10</sup>.



**Fig.6 Flow chart of current multiobjective optimization**

#### IV. Result

The number of feasible designs obtained here is 3,785 while 985 designs are non-dominated designs. Among them, eighty-three designs outperform the nominal design in all three design objectives. For example, one design increases 20 (kg) in final mass while maintaining its final velocity and minimum distance from the Sun.

The non-dominated solutions and the nominal design ( $M=1119.5$ ,  $V=10.2$ ,  $r_{min}=0.73$ ) are plotted in Fig. 7. This figure shows tradeoff among three design objectives. Figure 8 presents trajectories of final mass maximum, final velocity maximum, and minimum distance maximum designs.

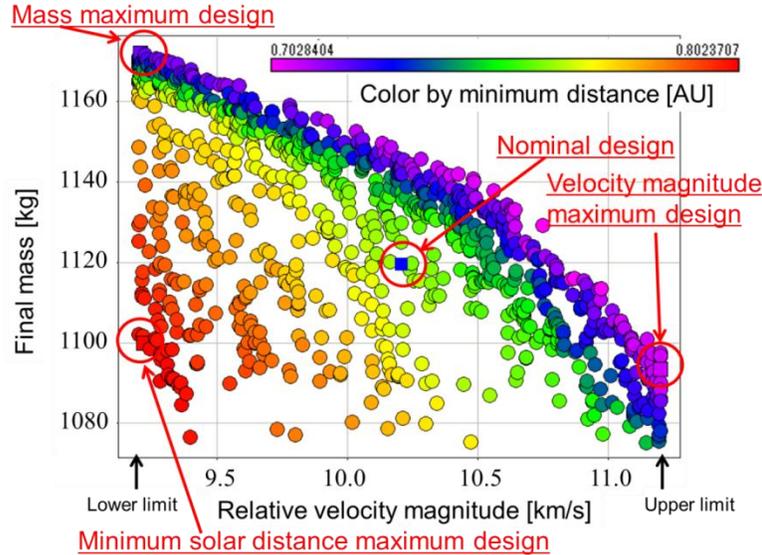


Fig.7 Distribution of non-dominated solutions.

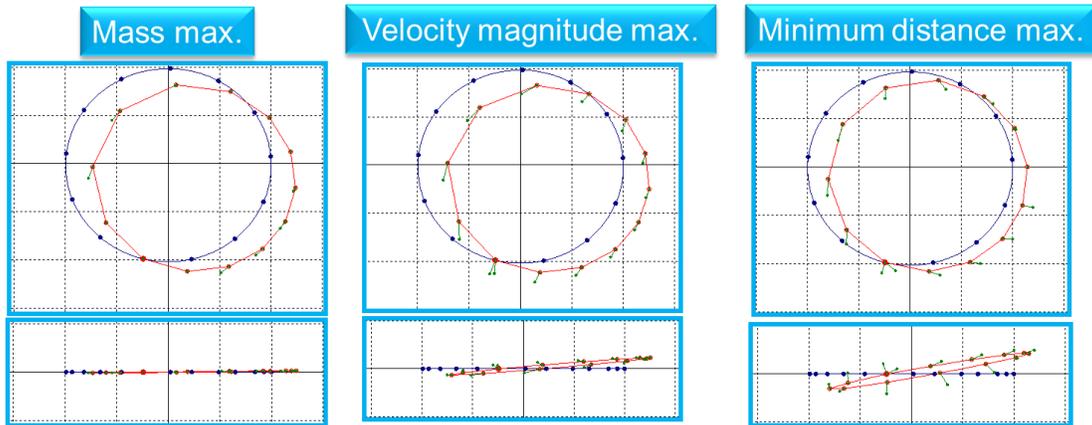
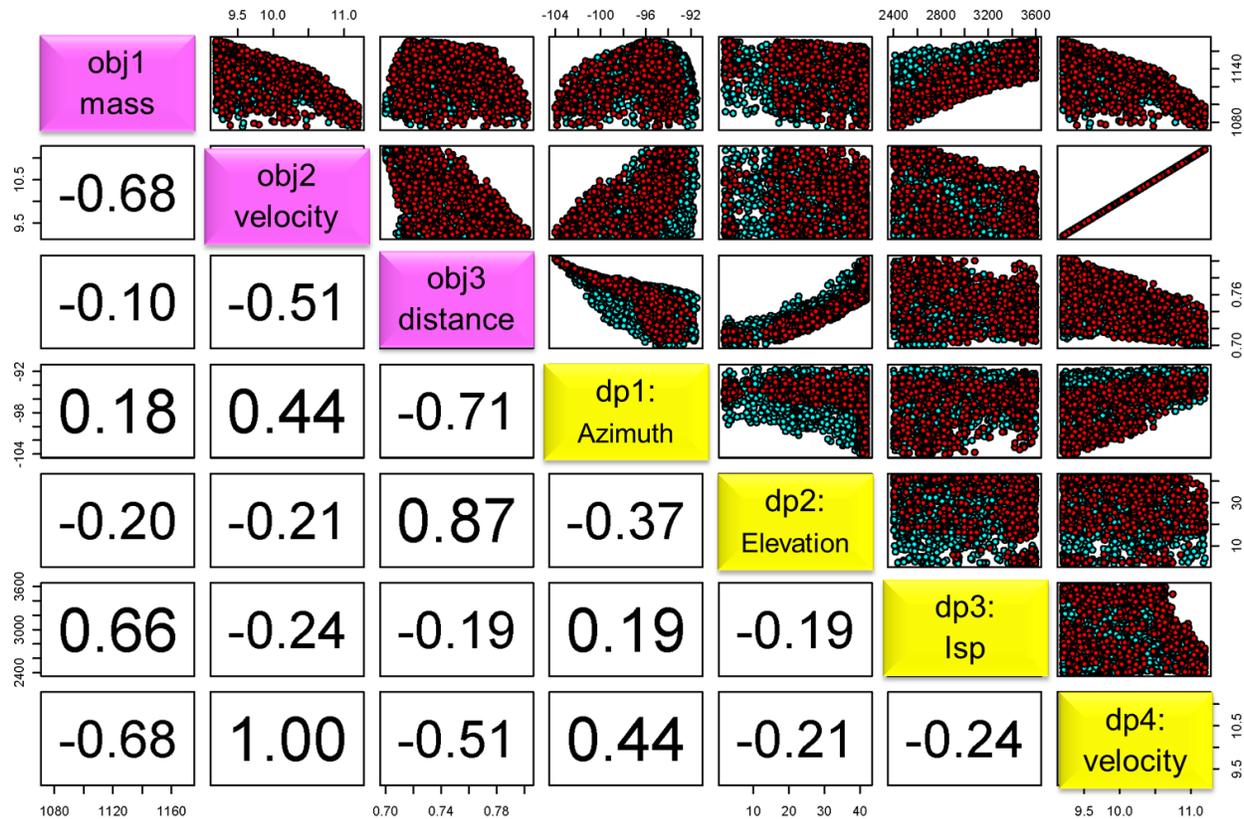


Fig.8 Trajectory and thrust profile of the solutions that maximize mass, velocity, or minimum distance.

Figure 9 is scatter plot matrix of non-dominated solutions and dominated solutions with correlation coefficients. This figure reveals feasible design space and the design space where the non-dominated solutions exist. This figure also shows sensitivity of each design parameter to each design objective. This figure includes much information for the trajectory design. For example, there is significant positive correlation between elevation angle and minimum distance from the Sun while the minimum distance from the Sun has negative correlation to the azimuth angle. This figure also presents new knowledge for trajectory design of SOLAR-C Plan A SEP option such as

- (1) elevation angles of the non-dominated solutions distribute in wide range of the design space independent of the final velocity value
- (2) there is a strategy to be a non-dominated solution with almost zero elevation angle (such solutions have large mass and velocity and small distance).



**Fig.9 Scatter plot matrix of non-dominated solutions (red) and dominated solutions (blue) with correlation coefficients.**

## VI. Conclusions

The capability of MODE approach to the trajectory design optimization of the SOLAR-C Plan A SEP option is demonstrated. The present approach revealed some important design knowledge such as tradeoff information among maximization of final mass, maximization of final velocity, and maximization of minimum distance from the Sun. Present study also showed possibility of simultaneous design of trajectory and spacecraft under MODE framework.

## References

- <sup>1</sup> Kawaguchi, J., Kawakatsu, Y., Morimoto, M., et al., On Ballistic Acquisition of Short Period Out-of-ecliptic Trajectories, *Advances in the Astronautical Sciences*, Vol. 135, pp. 577-588, 2009
- <sup>2</sup> Kawaguchi, J., SOLAR-C Ballistic Solar Polar Flight Option - Part-II via Venus Multiple Swingby -, *Proceedings of the 31st Workshop on Solar Science*, 2009
- <sup>3</sup> Kawakatsu, Y., Kuninaka, H. and Nishiyama K., Trajectory to the Orbit Largely Inclined with the Ecliptic Plane by way of Electric Propulsion Delta-v Earth Gravity Assist, *Advances in the Astronautical Sciences*, Vol. 134, pp. 801-812, 2009
- <sup>4</sup> Jeong, S., Chiba, K., and Obayashi, S., "Data Mining for Aerodynamic Design Space," *Journal of Aerospace Computing, Information, and Communication*, Vol. 2, No. 11, 2005, pp. 452-469.
- <sup>5</sup> Deb, K., *Multiobjective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, Ltd., Chichester, UK, 2001.
- <sup>6</sup> Chiba, K., Oyama, A., Obayashi, S., and Nakahashi, K., "Multidisciplinary Design Optimization and Data Mining for Transonic Regional-Jet Wing," *Journal of Aircraft*, Vol. 44, No. 4, 2007, pp. 110-1112.
- <sup>7</sup> Oyama, A., Okabe, Y., Fujii, K., Shimoyama, K., "Aerodynamic Multiobjective Design Exploration of a Flapping Airfoil Using a Navier-Stokes Solver," *Journal of Aerospace Computing, Information, and Communication*, Vol. 6, No. 3, 2009, pp. 1542-9423.
- <sup>8</sup> Tani, N., Oyama, A., and Yamanishi, N., "Multiobjective Design Optimization of Rocket Engine Turbopump Turbine," *Proceedings of the 5th International Spacecraft Propulsion Conference / 2nd International Symposium on Propulsion for Space Transportation [CD-ROM]*, 2008.
- <sup>9</sup> Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T., "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, Vol. 6, No. 2, 2002, pp. 182-197.
- <sup>10</sup> <http://www.iitk.ac.in/kangal/codes.shtml>