

# Aerodynamic Wing Optimization via Evolutionary Algorithms Based on Structured Coding

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## Abstract

Evolutionary Algorithms (EAs) based on structured coding have been proposed for aerodynamic optimization of wing design. Fractional factorial design is used to investigate interactions of the design variables to determine the appropriate coding structure for EAs in advance. The present EAs is applied to wing design problems where the wing shape is modeled using the parameter set for the extended Joukowski airfoils and the PERSEC airfoils. Aerodynamic optimizations of a transonic wing demonstrated that the structured coding for EAs is a promising approach to find a global optimum in real-world applications. The design results also confirm that the PERSEC is an efficient approach for transonic wing shape parameterization.

## 1. Introduction

Evolutionary Algorithms (EAs, for example, see [1]) are emergent numerical optimization algorithms modeled on mechanism of natural evolution by selection. One of the key features of EAs is that they search from multiple points, instead of moving from a single point unlike gradient-based methods. In addition, they require no derivatives or gradients of the objective function. These features lead to remarkable robustness in optimization and simplicity in coupling CFD codes together. Furthermore parallel efficiency will be extremely high by using a simple master-slave concept for function evaluations, if such evaluations consume most of CPU time. An examples is aerodynamic shape design optimization using CFD.

Thanks to these features, EAs are gaining popularity in various engineering fields (see [2], for example). As for aerodynamic designs, EAs have been applied to aerodynamic designs of two-dimensional shapes such as airfoils and turbine blades [3-5]. Even a three-dimensional wing design has been demonstrated in [6] by simplifying the geometry definition according to subsonic aerodynamics.

Application of EAs to practical aerodynamic optimization, however, may not be straightforward. Since such optimization problems usually require a large number of design parameters involving complex interactions each other, standard EAs would fail to find a globally optimum. Transonic wing design might be a typical case.

When EAs are used to solve an engineering optimization problem, complexity in the objective function distribution appears as interactions among design parameters. They are often referred as “epistases”, corresponding to the term used in biology. Therefore, if the epistases of the design parameters are identified in advance, a smoother landscape of the objective function distribution can be reproduced by rearranging encoding of the design parameters. However, an exhaustive search of epistases using such as full factorial design would require as many CFD analysis's as those required for EAs itself.

In [7], EAs using structured coding have been proposed and applied to a transonic wing shape design, where Fractional Factorial Design (FFD, [8]) was used to examine the epistasis of the design parameters. FFD is a statistical tool developed to gain needed information at the least expenditure of resources from a structured set of coherent tests. In the reference, although the structured coding improved the design performance, wing profiles were parameterized by the extended Joukowski transformation [9], which lately proven to be inadequate for precise geometry definitions, especially for transonic airfoil [10].

The objective of the present study is to apply an EA based on the structured coding to a three-dimensional wing shape design using a more precise airfoil parameterization technique. In this study, a sophisticated airfoil parameterization technique called “PERSEC” [11] will be used. The key concept of the PERSEC is that the choice of design parameters should be based on the flow structure around an airfoil to control the important aerodynamic features effectively. By applying the present EA to the wing shape design using the PERSEC, an efficient and robust EA-based transonic wing shape optimization tool will be developed.

In the present paper, first, application of the present EA to the aerodynamic wing shape design parameterized by the extended Joukowski transformation will be presented. Then the present EA will be applied to the same wing design problem where the wing shape is parameterized by the PERSEC. Finally, the results will be compared and discussed.

## 2. Approach

### 2.1 Geometry Modeling

The wing planform is taken from a typical transonic aircraft as Ref. 5. Wing profiles are parameterized by the extended Joukowski transformation or the PERSEC. Thus, an wing shape is defined by parameters for these airfoil shape parameterization techniques and twist angle  $\alpha$  given at five spanwise sections. The wing surface is linearly interpolated between the specified spanwise sections. In the following subsections, airfoil parameterization techniques are described.

#### The extended Joukowski transformation

The extended Joukowski transformation [9] can express various kinds of airfoils with small number of parameters by transforming a circle to an airfoil shape in the complex number plane with two consecutive conformal mappings as,

$$z' = z - \frac{\epsilon}{(z - \Delta)} \quad (1)$$

$$V = z' + \frac{1}{z'} \quad (2)$$

where  $\epsilon$  and  $\Delta$  are a complex parameter and a real parameter, respectively. Equation (2) is well known as the Joukowski transformation. And eq. (1) is a preliminary transformation before the Joukowski transformation. The airfoil shape is defined by 5 parameters: center of the circle  $Z_c$ , real part and imaginary part of  $\epsilon$ , and  $\Delta$ . An example of the extended Joukowski transformation is illustrated in Fig. 1. Instead of the raw parameters ( $Z_c$ ,  $\epsilon$ ,  $\Delta$ ), the present design parameters are given by  $(x_c, y_c, x_t, y_t, \Delta)$  where the center of the circle  $Z_c$  and the complex number  $\epsilon$  correspond to the position  $(x_c, y_c)$  and  $(x_t, y_t)$ , respectively. It is known that  $x_c$  and  $x_t$  are related to the airfoil thickness while  $y_c$  and  $y_t$  are related to the airfoil camber.

#### The PERSEC airfoils

Recently, an airfoil family “PARSEC” has been proposed to parameterize an airfoil shape [11]. Similar to 4-digit NACA series airfoils, the PERSEC parameterizes upper and lower airfoil surfaces using polynomials in coordinates  $X, Z$  as,

$$Z = \sum_{n=1}^6 a_n \cdot X^{n-1/2} \quad (3)$$

where  $a_n$  are real coefficients. Instead of taking these coefficients as design parameters, the PERSEC airfoils are defined by basic geometric parameters: leading-edge radius, upper and lower crest location including curvatures, trailing-edge ordinate, thickness, direction and wedge angle as shown in Fig. 2. These parameters can be expressed by the original coefficients  $a_n$  by solving simple simultaneous equations. Eleven design parameters are required for the PERSEC airfoils to define an airfoil shape in total.

The key concept of this parameterization is to keep number of the needed parameters as low as possible while controlling the important aerodynamic features effectively by selecting the design parameters based on the knowledge of transonic flows around the airfoil. Aerodynamic airfoil designs using EAs in [10] have proven that EA using the PERSEC succeeded in finding the best design among EAs using several airfoil parameterization techniques including a widely-used B-Spline curves.

### 2.2 Multiobjective Evolutionary Algorithms (MOEAs)

Most engineering problems require the simultaneous optimization of multiple, often competing criteria.

Such problems are called multiobjective or multicriteria problems. Unlike single objective optimization, the solution to this sort of problems is not a single point, but a family of points known as the Pareto-optimal set. Since EAs maintain a population of design candidates in parallel, application to multiobjective design problems is straightforward. EAs developed for multiobjective optimization problems are called Multiobjective Evolutionary Algorithms (MOEAs).

Generally, EAs consist of fitness evaluation of individuals, selection according to the fitness, crossover and mutation of mating pair's genes as illustrated in Fig. 3. When EAs are applied to engineering problems, an individual, fitness and genes correspond to a design candidate, objective function value(s) and design variables, respectively. In this study, the design variables are coded as real number strings since it is natural to use real numbers for optimization problems involving real parameters. Pareto-ranking method and fitness sharing [12] are combined into the present EA to obtain Pareto-optimal set in parallel. The best-N selection [13] is also incorporated, where the best N individuals are selected for the next generation among N parents and N children so that Pareto solutions will be kept once they are formed. Since the strong elitism is used, high mutation rate of 0.2 is applied and a random disturbance is added to the parameter in the amount up to  $\pm 20\%$  of the design space. Although several crossover techniques have been proposed, one-point crossover is applied to utilize the structured coding. The initial population is randomly created. Population size and maximum number of generations are set to 128 and 300, respectively. In total, present optimization requires 19264 CFD runs.

### 2.3 Fractional Factorial Design

A parametric study is often conducted by varying one parameter at a time or by trial and error for a limited number of parameters. However, such approaches only lead to incomplete knowledge for a large design space. An exhaustive search, in contrast, requires unacceptably large number of experiments and thus they are not suitable to real-world problems. For instance, a full factorial design of a design space of 10 parameters with 3 levels would require  $3^{10} = 59049$  experiments.

FFD, same times called experimental design, is a statistical approach that has been developed to gain needed information at the least expenditure of resources from a structured set of coherent tests. It reduces the required number of experiments by arranging the experiments according to the orthogonal array and estimating the effectiveness of the factors and their interactions by F-tests. These days, FFD is often used for screening experiment of the response surface method.

## 3. Results

Aerodynamic optimization of wing designs is demonstrated. The cruising Mach number is assumed to be 0.8. Airfoil thickness is constrained so that the maximum thickness is greater than 0.08 of the chord length. MOEAs search tradeoff solutions between maximization of  $C_L$  and minimization of  $C_D$  within one optimization. Aerodynamic performances are evaluated by the FLO-27 code, which is a conservative full-potential code developed by Jameson and Caughey [14]. The numbers of the required design parameters are 30 for the wing parameterized by the extended Joukowski transformation and 55 for the wing parameterized by the PERSEC.

### 3.1 Wing Design Using the extended Joukowski transformation

Prior to the design optimization, FFD is applied to analyze the epistases, i.e., the interactions of the design variables. Analysis of interactions of all design variables for the wing model, however, requires unacceptably large number of CFD runs even with the FFD. Therefore, the design variables are grouped into spanwise variations of the airfoil shape parameters and the twist angle. Factors examined are these spanwise distributions and their two-factor interactions except for those related to the twist angle  $\alpha$ . Three types of spanwise variations are considered as levels: no variation, linear increase from root to tip, and vice versa. Examined responses are  $C_L$  and  $C_D$  of the wing. Only to account for positive responses in aerodynamic performance (increase in  $C_L$  and decrease in  $C_D$ ), following two functions are introduced:

$$F1 = \max ( C_L - C_{L0} , 0 ) \quad (4)$$

$$F2 = - \min ( C_D - C_{D0} , 0 ) \quad (5)$$

where  $C_{L0}$  and  $C_{D0}$  are those of a wing having a constant airfoil section along the spanwise direction.

Since this is the case of six factors, ten interactions, and three levels, 81 CFD runs are conducted according to the  $L_{81}(3^{40})$  orthogonal array. Then the results are statistically analyzed by F-tests. Figure 4

shows the F values of the examined factors and interactions. The solid and broken lines are critical F values with 1% and 5% statistical risks, respectively. A factor or an interaction that has F value more than these critical values is judged effective. While every single factor is effective on both F1 and F2, nothing but  $x_c, x_t$  and  $y_c, y_t$  appears effective among the examined interactions. This result is consistent with the fact that  $x_c$  and  $x_t$  are related to the airfoil thickness while  $y_c$  and  $y_t$  are related to the airfoil camber line.

To make use of identified interactions of the design variables,

- 1) Structured coding is introduced by considering each spanwise distribution of the airfoil parameters and the twist angle as strings of genes instead of conventional sequential coding where all design variables are coded as a single string.
- 2) One-point crossover is applied to each string where the same gene site is selected for each interactive design parameter sets ( $x_c, x_t$ ) and ( $y_c, y_t$ ), at the probability)

$$P = 0.1 + 0.7(\min(1, \text{generation} / 50)) \quad (6)$$

Figure 5 illustrates the proposed structured coding for the present wing shape modeling. The broken lines in the figure show how one-point crossover is applied to the present structured coding. This crossover enables that the genes of the identified parameter sets, such as ( $x_c, x_t$ ) and ( $y_c, y_t$ ) are exchanged together to utilize efficiently the effects of the interactions between them.

To validate advantage of the present approach, design optimization is demonstrated using the present EA and the design results are compared with that of the EA with the sequential coding where one-point crossover is applied to each spanwise distribution of the design parameters but each crossover gene site is selected independently as illustrated in Fig. 6. Figure 7 shows the Pareto optimal solutions indicating the tradeoff between maximization of  $C_L$  and minimization of  $C_D$ . Solid and hollow points show the resulting Pareto fronts obtained from the sequential and structured codings, respectively. This figure indicates that the present EA with the structured coding have better Pareto solutions in high  $C_L$  region.

### 3.2 Wing Design Using the PERSEC

Now, the present approach is applied to wing design using the PERSEC. First, the epistases of the parameter sets for the PERSEC airfoils are analyzed by FFD. The factors to be examined are  $r_{LE}, X_{UP}, Z_{UP}, Z_{XXUP}, X_{LO}, Z_{LO}, Z_{XXLO}, \alpha_{TE}, Z_{TE}$  and their two-factor interactions on F1 and F2. The wedge angle at the trailing edge and its interactions are neglected since the wedge angle is primary determined by the structural strength. Also, interactions of  $r_{LE}Z_{TE}, r_{LE}\alpha_{TE}$  and  $r_{LE}Z_{XXLO}$  are disregarded. Consequently, 42 factors are examined. And FFD is conducted according to the  $L_{729}(3^{364})$  design template. Number of CFD runs required for this epistasis analysis is reduced from  $3^9 = 19683$  (full factorial design) to  $3^6 = 729$ .

Figure 8 shows the result of the F-tests. Interactions effective in both  $C_L$  and  $C_D$  are illustrated with bold lines in Fig. 9. Since these figures indicate complicated interactions among the design variables, especially,  $Z_{UP}, Z_{LO}$ , and  $Z_{TE}$ , it seems difficult to construct a structured coding for these design variables. Therefore, new parameters  $Z_C$  and  $Z_H$  are introduced instead of  $Z_{UP}$  and  $Z_{LO}$  as;

$$Z_C = (Z_{UP} + Z_{LO}) / 2 \quad (7)$$

$$Z_H = (Z_{UP} - Z_{LO}) \quad (8)$$

where  $Z_C$  and  $Z_H$  correspond to airfoil camber and thickness, respectively. Using these parameters, interactions are greatly simplified as shown in Fig.10. According to this result, a structured coding for the spanwise distributions of airfoil parameters is introduced.

The design result obtained by the EA using this structured coding is compared with that obtained by the EA using the sequential coding. Figure 11 compares Pareto fronts obtained from the sequential coding of the original PERSEC airfoils and the structured coding using  $Z_C$  and  $Z_H$ . Similar to the previous section using the extended Joukowski airfoils, advantage of the structured coding is observed in high  $C_L$  region. Compared with Fig.7, this figure also shows that Pareto front of the PERSEC airfoils is superior to that of the extended Joukowski airfoils.

### 4. Conclusion

EAs based on structured coding have been proposed for aerodynamic optimization of wing design. The coding structure for EA was developed according to the epistasis analyzed by FFD. The present approach was applied to wing design problems where the wing shape is modeled using the parameter sets defined by the extended Joukowski airfoils and by the PERSEC airfoils. Aerodynamic optimizations of a transonic

wing demonstrated that the structured coding for EAs is a promising approach to find a global optimum in practical applications. The design results also confirm that the PERSEC is an accurate technique for transonic wing shape parameterization. The improved Pareto front is obtained by EA based on the proposed structured coding using the PERSEC.

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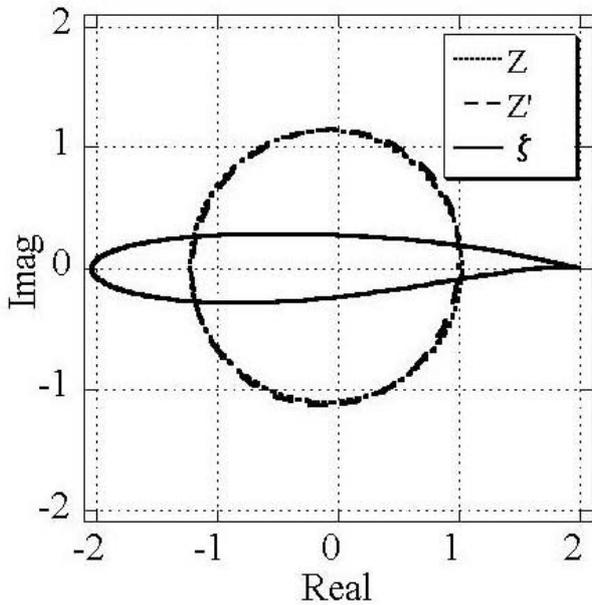


Figure 1. Example of the extended Joukowski transformation.

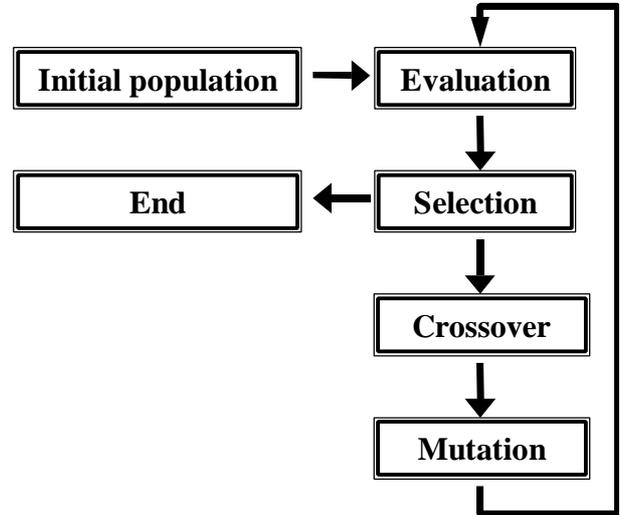


Figure 3. Flowchart of Evolutionary Algorithms.

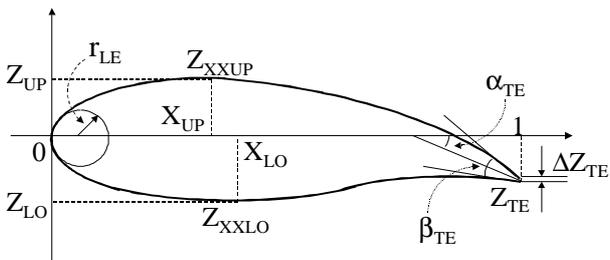


Figure 2. Design parameters for the PERSEC airfoils.

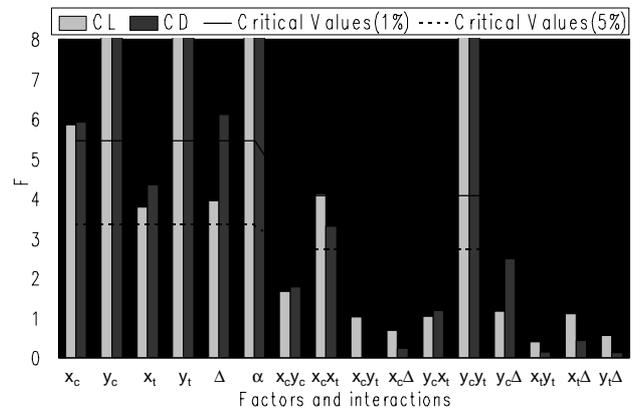


Figure 4. Effectiveness of factors and their interactions for the extended Joukowski transformation.

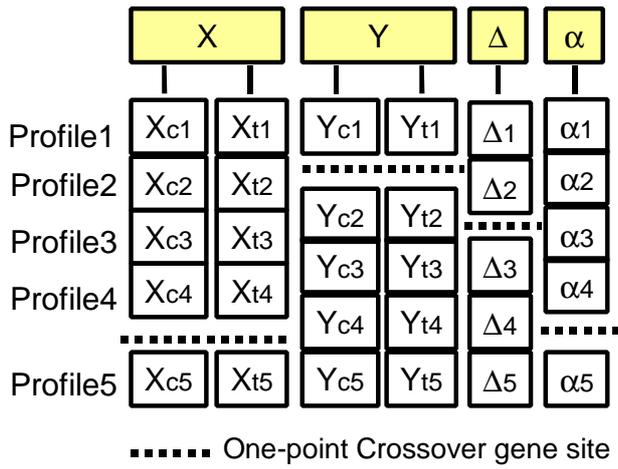


Figure 5. Structured coding for the extended Joukowski transformation.

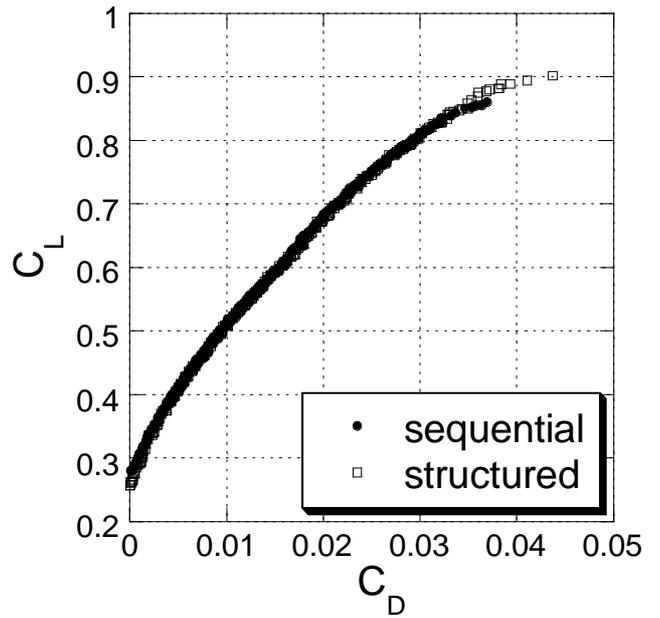


Figure 7. Comparison of Pareto fronts for Sequential and structured coding techniques.

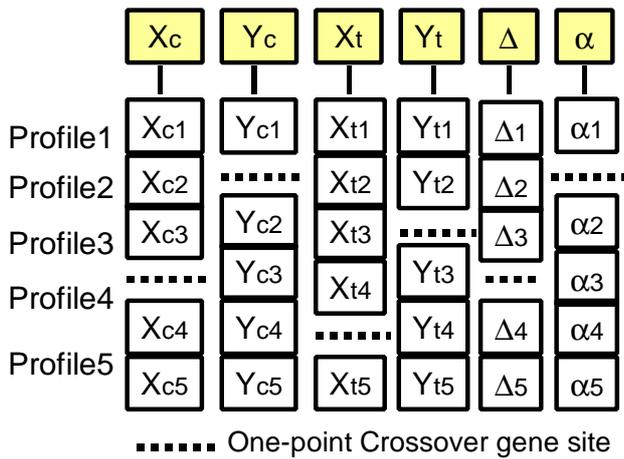


Figure 6. Sequential coding for the extended Joukowski transformation.

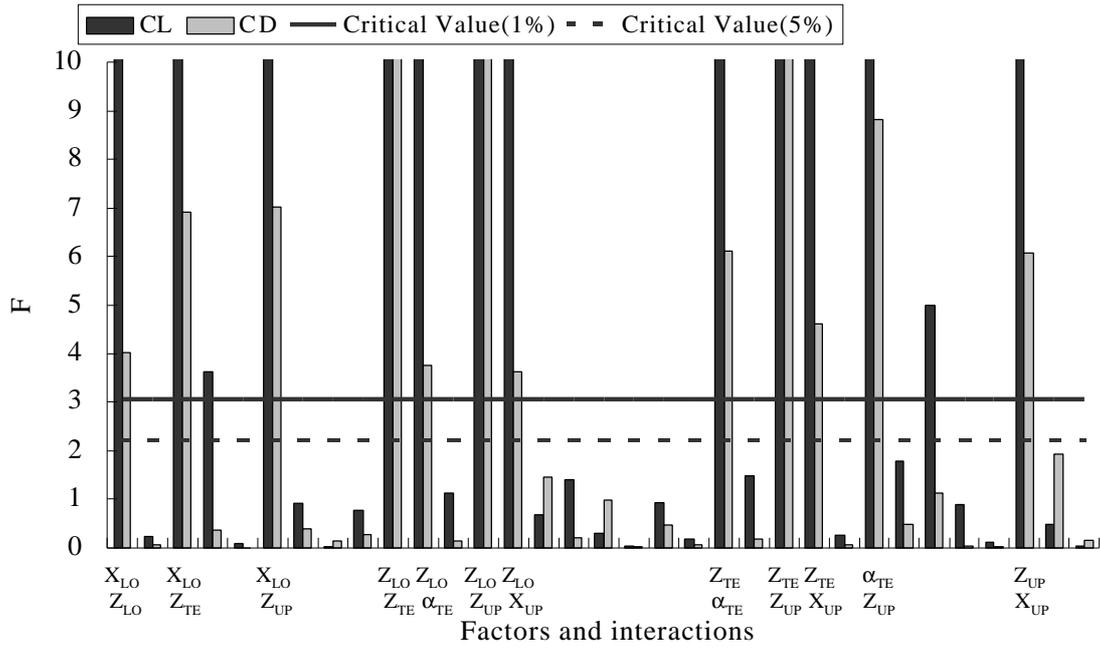


Figure 8. Effectiveness of factors and their interactions for the PERSEC.

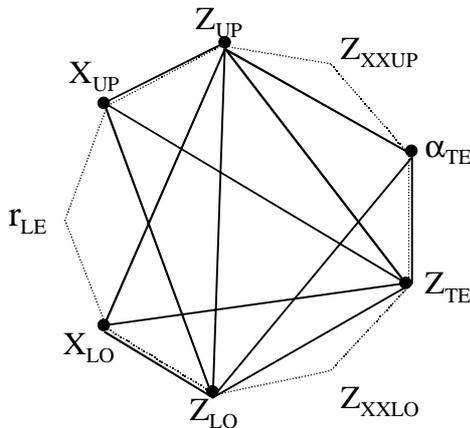


Figure 9. Effective interactions of the original PERSEC.

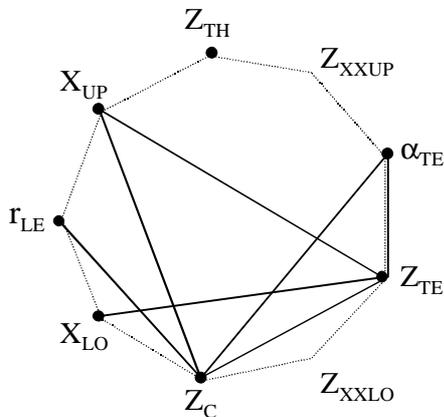


Figure 10. Effective interactions of the modified PERSEC.

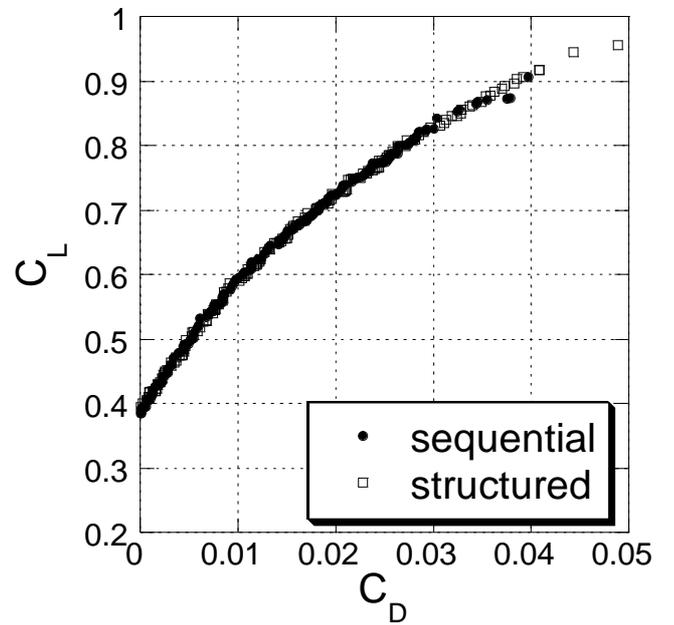


Figure 11. Comparison of Pareto fronts for sequential and structured coding techniques.

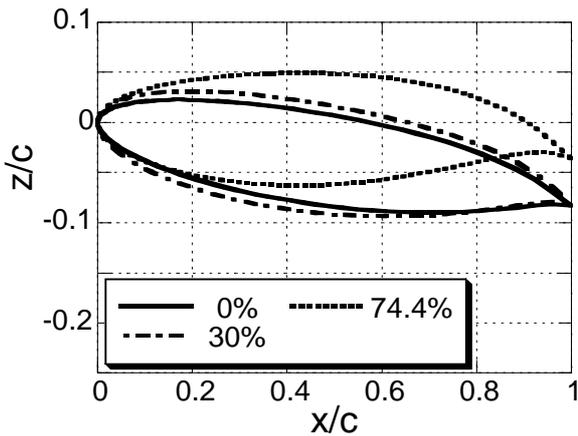


Figure 12. Wing profiles of the design obtained by the EA with sequential coding and the extended Joukowski parameterization.

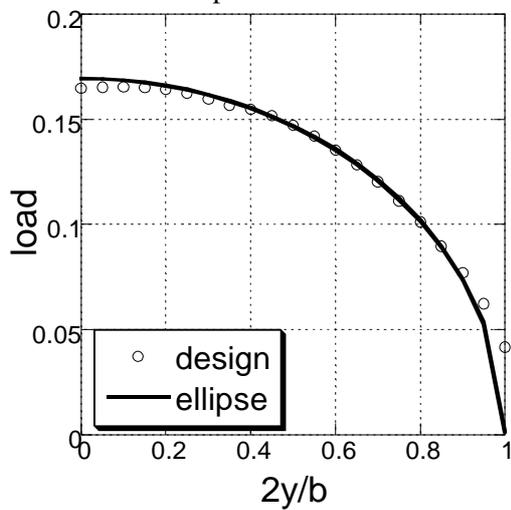


Figure 13. Spanwise load distribution of the design obtained by the EA with sequential coding and the extended Joukowski parameterization.

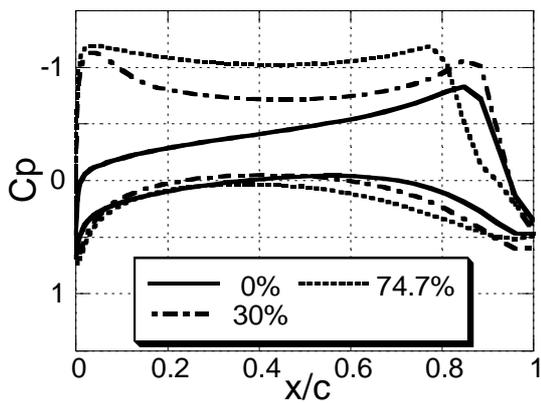


Figure 14. Cp distribution of the design obtained by the EA with sequential coding and the extended Joukowski parameterization.

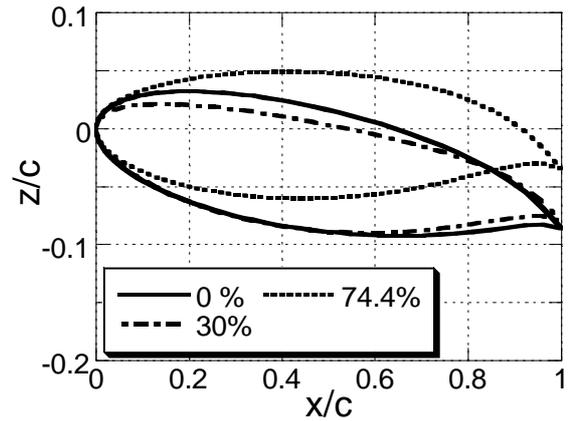


Figure 15. Wing profiles of the design obtained by the EA with structured coding and the extended Joukowski parameterization.

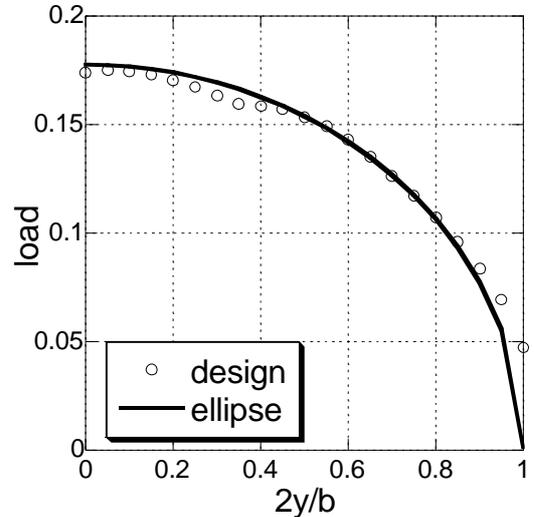


Figure 16. Spanwise load distribution of the design obtained by the EA with structured coding and the extended Joukowski parameterization.

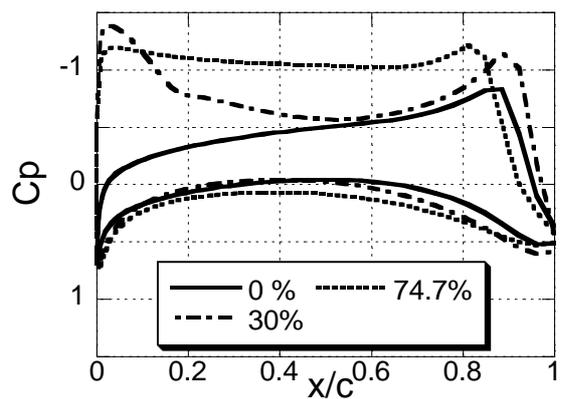


Figure 17. Cp distribution of the design obtained by the EA with structured coding and the extended Joukowski parameterization.

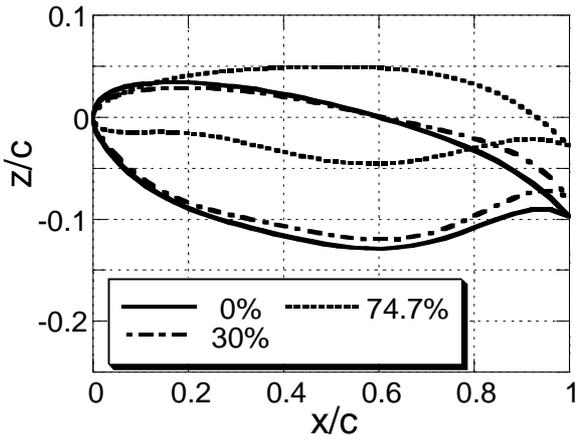


Figure 18. Wing profiles of the design obtained by the EA with sequential coding and the PERSEC parameterization.

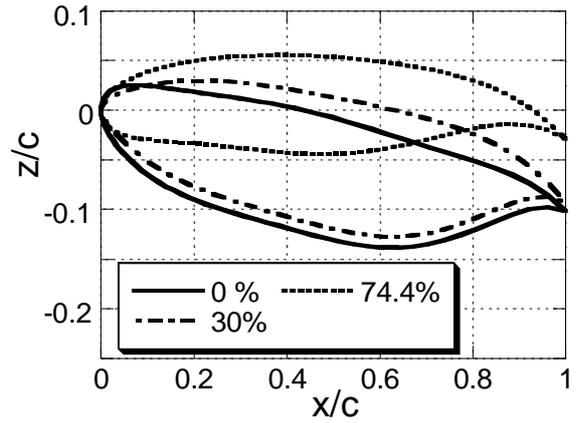


Figure 21. Wing profiles of the design obtained by the EA with structured coding and the PERSEC parameterization.

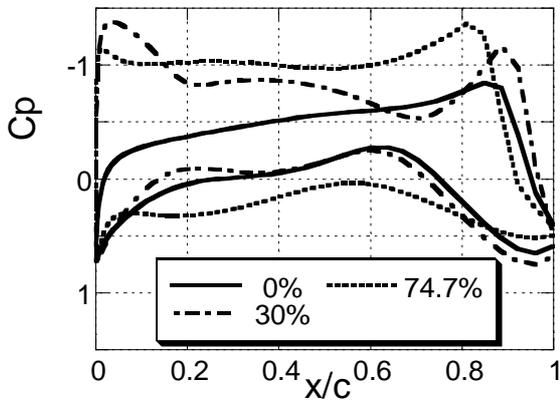


Figure 19. Cp distribution of the design obtained by the EA with sequential coding and the PERSEC parameterization.

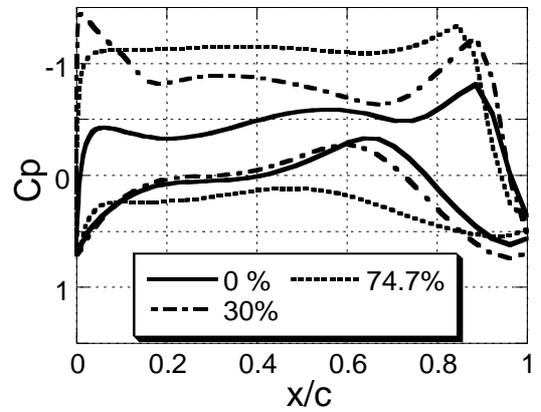


Figure 22. Cp distribution of the design obtained by the EA with structured coding and the PERSEC parameterization.

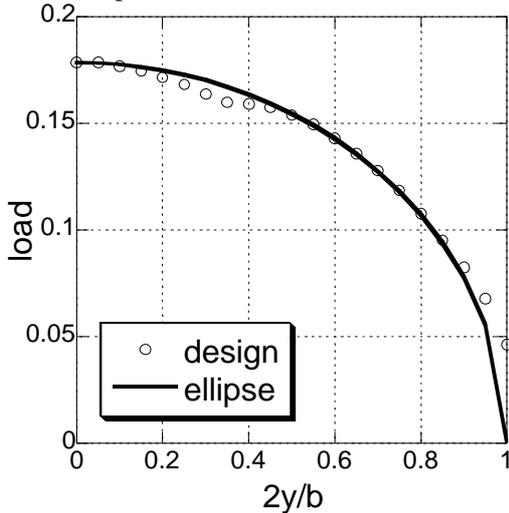


Figure 20. Spanwise load distribution of the design obtained by the EA with sequential coding and the PERSEC parameterization.

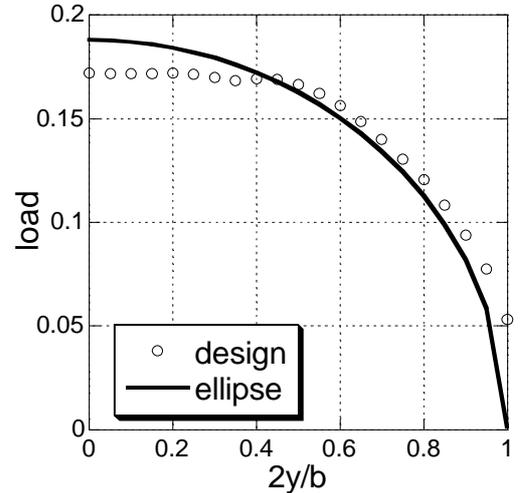


Figure 23. Spanwise load distribution of the design obtained by the EA with structured coding and the PERSEC parameterization.

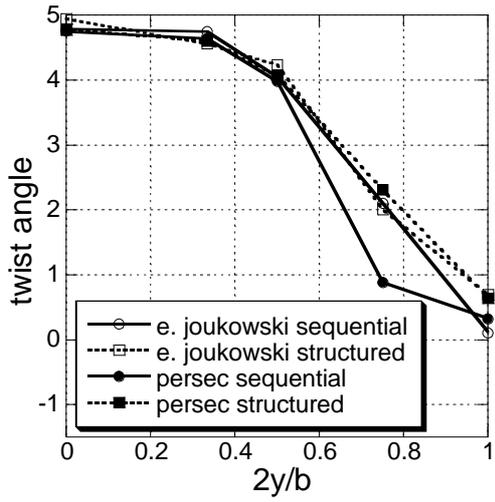


Figure 24. Comparisons of twist angle distributions of the designed wings.

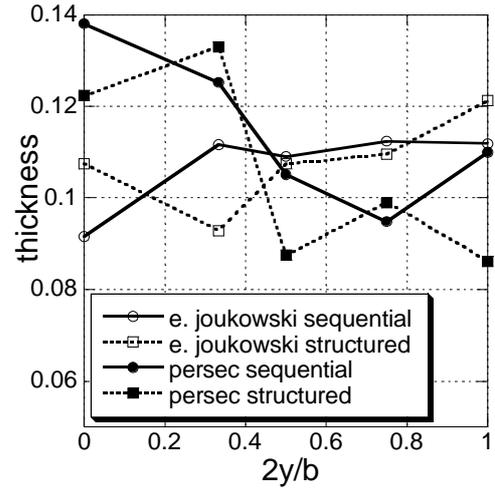


Figure 25. Comparisons of thickness distributions of the designed wings.