Euler/Navier-Stokes Optimization of Supersonic Wing Design Based on Evolutionary Algorithm

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This paper presents aerodynamic shape optimization of a supersonic wing for supersonic civil transportation (SST) using an Evolutionary Algorithm (EA) coupled with an Euler/Navier-Stokes code. To overcome enormous computational time necessary for the design, aerodynamic evaluations are parallelized on Numerical Wind Tunnel (NWT) at National Aerospace Laboratory, a parallel vector machine with 166 processing elements. Parallelization of function evaluations in EA is straightforward and its performance is extremely good since most of computational time is used by flow computations. The design result indicates that the present EA successfully minimizes both the induced drag and the volume wave drag in the given design space.

1. INTRODUCTION

Application of numerical optimization to aerodynamic design is a difficult task. In [1], it was reported that distribution of the objective function could be extremely rough even in a simplified problem. In addition, function evaluations using a Computational Fluid Dynamics (CFD) code, especially an Euler or Navier-Stokes code, are very expensive. Therefore, both an optimization algorithm with high parallel efficiency and a powerful parallel computer are required to accomplish aerodynamic optimization.

Among optimization algorithms, Evolutionary Algorithms (EAs, for example, see [2]) are emergent algorithms, which have recently been applied to aerodynamic design problems [3-5]. EAs are modeled on the mechanism of the natural evolution that consists of evaluation of fitness of population, selection according to fitness, crossover and mutation of mating pair's genes. When EAs are applied to numerical optimization, fitness, population and genes usually correspond to objective function value, design candidates and design variables, respectively. The flowchart of EAs is illustrated in Figure 1.

EAs have captivated many designers and researchers with their robustness as well as their high parallel efficiency. These characteristics originate in the following features of EAs:

1) EAs search from multiple points, instead of moving from a single point.
2) No derivatives or gradients of the objective function are required.
3) Many design candidates can be evaluated in parallel during each iteration.

Parallel efficiency becomes very high by using a master-slave concept for function evaluations, if such evaluations consume most of CPU time. Aerodynamic optimization using CFD will be a typical case.

Numerical Wind Tunnel (NWT, used by winners of IEEE's 1995 and 1996 Gordon Bell Prize for performance) is a MIMD parallel computer with 166 vector-processing elements (PEs) located at National Aerospace Laboratory in Japan. Its total peak performance is about 280 GFLOPS and the total main memory capacity is 45GB. The peak performance and main memory of each PE are 1.7 GFLOPS and 256 MB, respectively. Therefore, NWT has enough power for aerodynamic optimization using EAs coupled with Euler/Navier-Stokes evaluations.

The purpose of this study is to examine the feasibility of supersonic wing design optimization using EA coupled with Euler/Navier-Stokes computation. Grid generation and flow calculation of each design candidate are distributed to 64 Pes, while EA operators are assigned to the master computer because their CPU time is negligible. Airfoil sections of design candidates are represented by the extended Joukowski transformation [6], which has been developed for subsonic airfoil design.

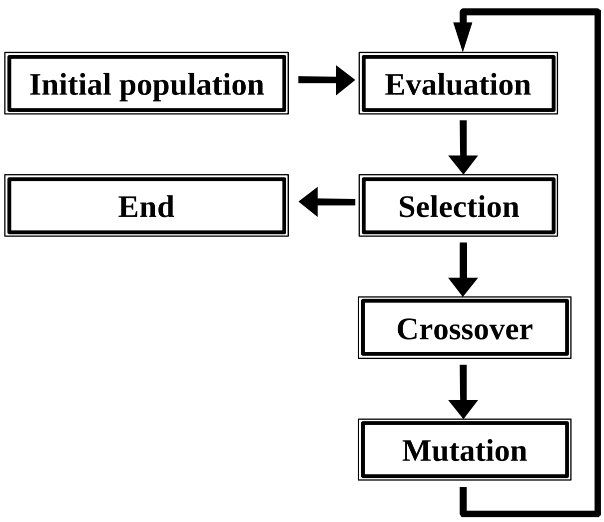

Figure 1. Flowchart of a typical Evolutionary Algorithm.

2. FORMULATION OF OPTIMIZATION PROBLEM

In this study, an aerodynamic shape of a supersonic wing is optimized at the supersonic cruise design point. The cruising Mach number is set to 2.3. Purpose of the present study is to maximize lift-to-drag ratio (L/D) maintaining substantial lift coefficient (C_L) and wing thickness. The optimization problem is defined as:

Objective function to be maximized: L/D

Constraints: C_L = 0.1, thickness-to-chord (t/c) > 0.35

Constraints are usually enforced by a penalty function. However, such a penalty may reduce feasible design space. Therefore, the lift constraint is satisfied by changing the geometric angle of attack at wing root α_{root} so that C_L becomes 0.1 based on the fact that the lift coefficient varies linearly:

$$(\alpha_{root})_{CL=0.1} = \left(\frac{(C_L)_{SPECIFIED} - (C_L)_{\alpha=\alpha 1}}{(C_L)_{\alpha=\alpha 2} - (C_L)_{\alpha=\alpha 1}} \right)(\alpha_2 - \alpha_1) + \alpha_1 \quad (1)$$

where α_1 and α_2 are set to 3 and 5 degrees, respectively. Two extra flow evaluations are necessary for this approach.

The aerodynamic performance is evaluated by using an Euler/Navier-Stokes code. This code employs TVD type upwind differencing [7], the LU-SGS scheme and the multigrid method [8].

Airfoil sections of design candidates are generated by the extended Joukowski transformation. It transforms a circle Z_0 to various kinds of airfoils in the complex number plane by two consecutive conformal mappings as,

$$Z_0 = re^{i\theta} + Z_c \quad (2)$$

$$Z_1 = Z_0 - \varepsilon / (Z_0 - \Delta) \quad (3)$$

$$Z = Z_1 + 1 / Z_1 \quad (4)$$

here Z_c, Z_0, Z_1, Z, and ε are complex numbers and Δ, r, and θ are real numbers, where r is determined so that Z_0 passes the origin of the coordinate axes. This transformation is therefore defined by Z_c, ε, and Δ.

Instead of the raw design variables (Z_c, ε, Δ), the present design variables are given by five parameters (x_c, y_c, x_t, y_t, Δ) where a position (x_c, y_c) corresponds to the center of the unit circle Z_0, the complex number ε corresponds to (x_t, y_t), and Δ is the preliminary movement in the real axis. It is known that x_c, x_t, and Δ are related to the airfoil thickness while y_c and y_t are related to the airfoil camber line.

Planform is assumed to be a double-delta wing similar to NAL scaled supersonic experimental airplane (Fig. 2). Airfoil sections defined by these extended

Joukowski parameters and the twist angle will be given at eight span sections, of which spanwise locations are also treated as design variables except for the wing root and tip locations. Wing geometry is then interpolated in spanwise direction by using the second-order Spline interpolation.

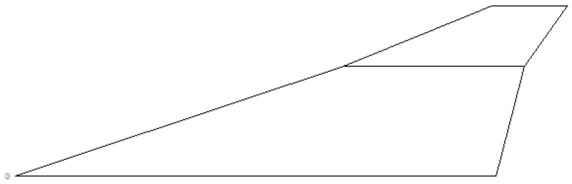

Figure 2. Wing planform.

3. OPTIMIZATION USING EA

In the present EA, design variables are coded in finite-length strings of real numbers corresponding to the five Joukowski transformation parameters, the twist angle, and their spanwise locations. The population size is kept at 64 and the initial population is created randomly within the present design space shown in Table 1. Fitness of an individual is determined by its rank among the population based on its L/D. Selection is performed by the stochastic universal sampling [9] coupled with the elite strategy. Ranking selection is adopted since it maintains sufficient selection pressure throughout the optimization. Then the offspring (the new design candidates) are produced applying one-point crossover [2] and evolutionary direction operator [10] half-and-half to the mating pool (selected design candidates). During the reproduction process, mutation takes place at a probability of 20% and then adds a random disturbance to the corresponding gene in the amount up to ±10% of each parameter range.

Table 1. Parameter ranges of the design space

Design variable	x_c	y_c	x_t	y_t	Δ	α
Upper-bound	-0.01	0.04	1.030	0.04	0.8	0 deg.
Lower-bound	-0.07	0.00	1.002	-0.02	0	-8 deg.

To reduce the wall clock time necessary for this optimization, evaluations using the Euler/Navier-Stokes code are distributed to 64 PEs of NWT. Since the CPU time used for EA operators are negligible, turnaround time becomes almost 1/64. While each CFD evaluation takes about one hour of CPU time (for three Euler evaluations) on the slave PE, the EA operators take less than one second on the master PE.

4. RESULTS

Since the wing planform is fixed and the viscous drag primary depends on the planform area, inviscid calculations are used for present evaluations. Therefore, the total drag evaluated here consists of the volume wave drag, the lift dependent wave drag and the induced drag. Among the three drag components, the lift dependent wave drag primary depends on the planform. Therefore, a design that achieves the minimum volume wave drag and the minimum induced drag will ensure the feasibility of the present approach.

The optimization history of the present EA is shown in Fig. 3 in terms of C_D. The design has drag coefficient of 77.7 counts and therefore L/D of 12.83. Since the evaluation takes about one hour per generation, the optimum is obtained in 50 hours.

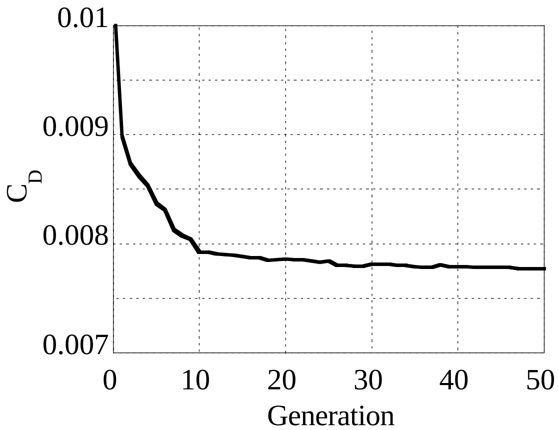

Figure 3. Optimization history.

Figure 4 compares the spanwise loading distribution of the designed wing with a parabola which is known to give the minimum induced drag when the structural constraint is considered [6]. The parabolic load distribution indicates the design achieves the minimum induced drag.

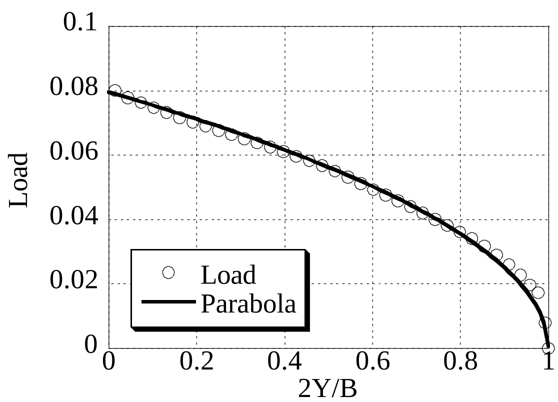

Figure 4. Spanwise load distribution of the designed wing.

The optimized airfoil sections at the 0%, 33% and 66% spanwise locations and the corresponding pressure distributions are shown in Figs. 6 and 7, respectively. The designed wing increases the camber toward the wing tip to increase c.. This helps to yield the parabolic load distribution and thus to achieve the minimum induced drag. On the other hand, the airfoil thickness becomes thinner as much as possible in the given design space to minimize the volume wave drag as expected. The plot is not shown here, since the thickness is simply 3.5% to the chord.

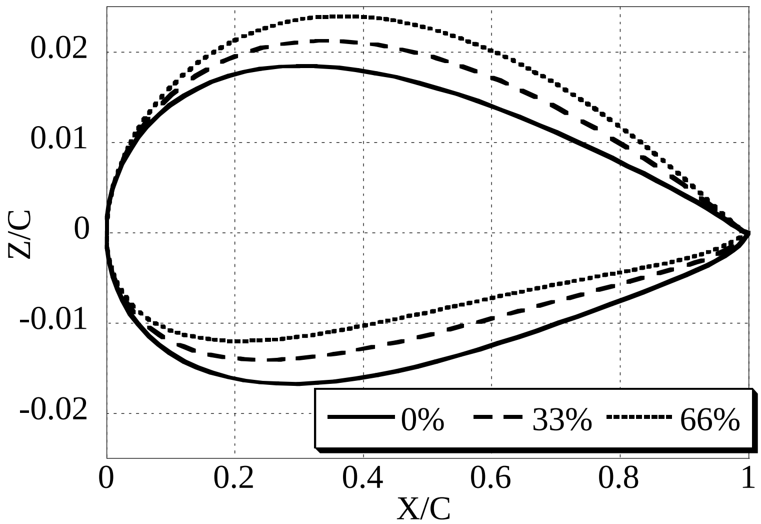

Figure 5. Designed airfoil sections.

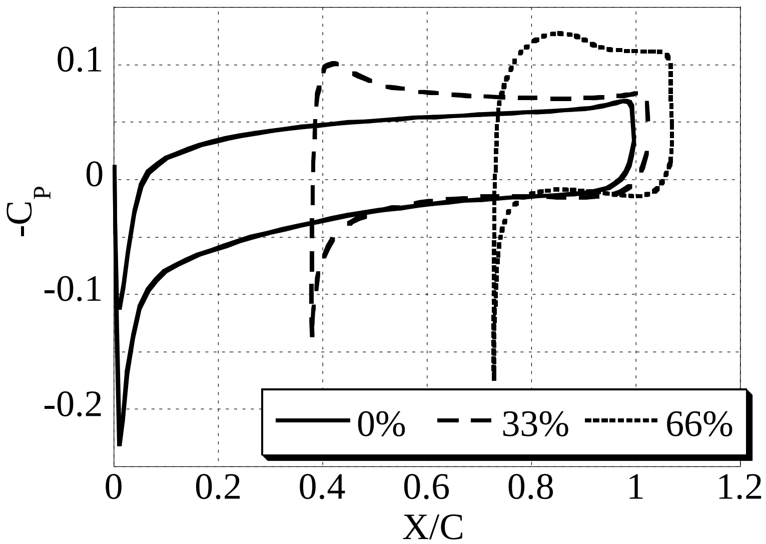

Figure 6. Corresponding C_P distributions.

Figure 7 illustrates the spanwise twist angle distribution and its control points of the designed wing. Geometric angle of attack is set to 5.63 degrees to have C_L of 0.1. Remarkably, three control points are located near the kink at the 60% spanwise location so that the wing twist reduces from 2.5 degrees to 0.5 degree rapidly. The spanwise twist angle distribution varies drastically here while the spanwise load distribution shown in Fig. 4 is surprisingly smooth. Resultant pressure contours on the upper surface of the wing is depicted in Figure 8.

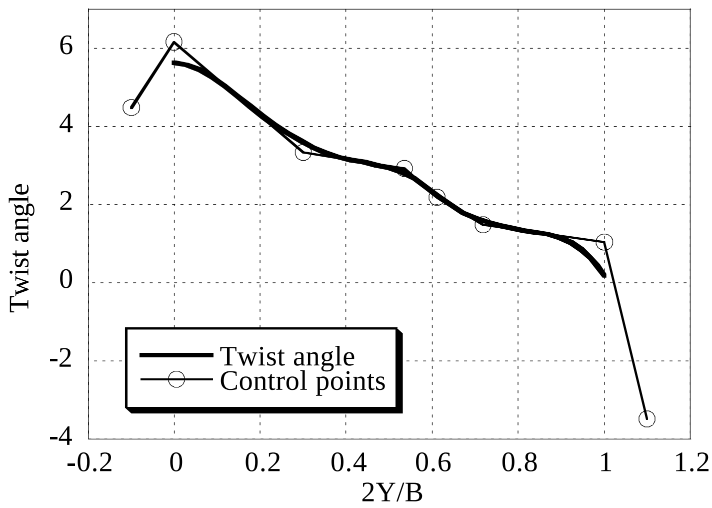

Figure 7. Spanwise twist angle distribution and its control points.

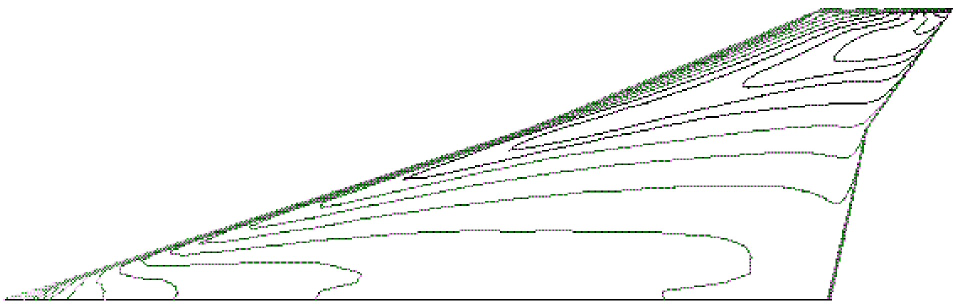

Figure 8. Pressure contours on the upper surface of the wing.

5. CONCLUSION

An EA coupled with an Euler/Navier-Stokes code has been applied to supersonic wing shape design. To overcome enormous computational time necessary for the optimization, aerodynamic evaluations are distributed to the PEs of NWT. Parallelization of EA on NWT is straightforward and its performance is extremely good in reducing the turnaround time.

The optimum design obtained from the present approach yields both the minimum induced drag and the minimum volume wave drag in the given design space. This indicates the feasibility of the present approach for aerodynamic design of SST. In addition, the present study indicates outstanding features of supersonic wing design compared with conventional transonic wing design as follows:

1) Warp geometry based on camber line and twist angle distributions plays a more important role than thickness distribution.
2) The structural constraint is found important to determine wing thickness and thus more practical structural constraint will be required.

Future work will be made towards multi-point design for supersonic and transonic cruises.

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