A Proposal of Airfoil Parameters Providing Good Correlation with Aerodynamic Performance

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A new parameter set to define the airfoil geometry for transonic aerodynamic airfoil shape optimization is proposed. Pareto-optimal solutions of the optimization problem are analyzed using correlation coefficients and scatter plot matrices. The results show that the proposed parameter set contains more parameters that have strong influence on lift and/or drag coefficients than the coordinates of B-Spline curve control points. The performance in this respect is similar to PARSEC parameters. In addition, the new method contains more parameters that show good correlation with both the lift and drag coefficients at the same time providing insight on the tradeoffs involved with transonic airfoil design.

1. Introduction

Many different sets of parameters are available to describe the geometry of an airfoil. Before advanced CFD simulations existed, parameterized airfoils shapes were used for large scale wind tunnel research. This research resulted in databases of airfoil geometries and their properties allowing designers to select the most suitable airfoil for their application. When using computational design optimization it is also important to have a limited set of parameters to define the airfoil's shape. A too large set of parameters would lead to excessive computation time to search the design space.

Both applications mentioned above have one aspect in common: the best solution that can be obtained depends on the parameterization. It is impossible to find solutions that the parameter set is unable to describe. This raises the following question: what geometrical features of an airfoil are important for its aerodynamic performance? Moreover knowledge of the influence of different parameters on the performance of an airfoil and the different tradeoffs involved can greatly improve the designer's understanding of the problem at hand.

To address these issues the Multi-Objective Design Exploration (MODE) method has recently gained interest. This method combines multi-objective optimization with data mining. Using MODE information about correlations and tradeoffs between objective functions, constraints and design parameters can be obtained. The MODE method has also been applied to problems related to aerodynamics [1].

For the optimization problem genetic algorithms based on combinations and mutations of previous designs have proven to be successful [2]. To perform data-mining various statistical and visualization methods are available such as Self Organizing Maps (SOM) [3], Analysis of Variance (ANOVA) and Scatter Plot Matrices (SPM) [4].

Objective of the present study is to analyze the correlation of design parameters describing a transonic airfoil shape with aerodynamic performance (lift and drag coefficients) and to propose a new design parameter set. To achieve this goal, first Pareto-optimal airfoil shapes are obtained for a transonic aerodynamic multi-objective optimization problem (lift coefficient maximization/drag coefficient minimization) using B-Spline curve based airfoil shape representation. Then, values of PARSEC parameters [5] and the proposed parameter set are obtained from the Pareto-optimal solutions. Finally, correlations of the B-Spline parameters, PARSEC parameters, and proposed parameter set with the design objectives are compared using correlation coefficients and scatter plot matrices.

2. Optimization Problem Formulation and Multi-Objective Design Exploration

The MODE method used for aerodynamic airfoil design consists of a number of steps. The first step is to select a set of parameters to define the airfoil geometry.

In this research 9 B-Spline control points describe the shape of the airfoil (see Fig. 1). Two control points are located at the trailing edge, these points function as start- and endpoint for the curve. Since each control point has a x and y coordinate and the leading and trailing edge are fixed at (x,y)=(0,0) and (1,0) this results in a total of 12 parameters. To deal with the boundary conditions at the start- and endpoints two additional control points are linearly extrapolated behind the trailing edge, where the first derivatives are set equal to those of the extrapolated lines.

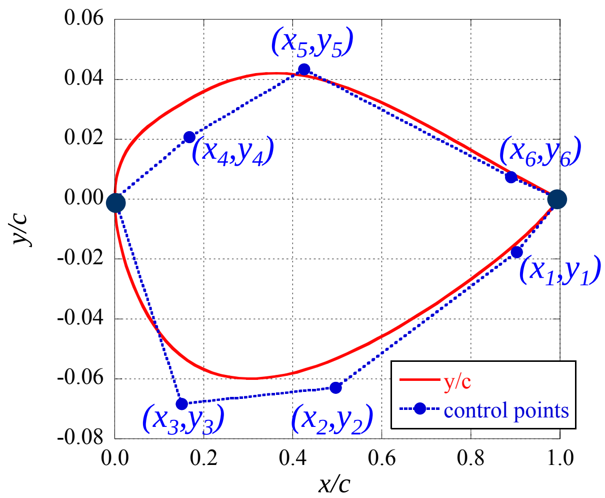

Fig. 1 B-Spline control points

Then the objectives and constraints have to be chosen. The constraints used in this research can be found in Tab. 1. In addition to these constraints two objectives are used: maximization of lift coefficient *Cl* and minimization of drag coefficient *Cd*. These coefficients represent the aerodynamic performance of the individual designs.

Tab. 1 Constraints on the individual solutions

Property	Constraint
Minimum thickness	$\geq 10\%$ of the chord length
Cl	≥ 0

Next the first airfoil shapes that function as a starting point for the multi-objective evolutionary algorithm (MOEA) are determined. The flow fields around these airfoils are simulated using CFD. Solutions that have not sufficiently converged are excluded. Then the non-dominated solutions are selected and a Pareto ranking is assigned to the different airfoils based on the amount of airfoils that dominate it. A solution is classified as non-dominated when there are no solutions that perform better on at least one design objective and perform at least equal on the other objectives. The set of all non-dominated solutions is called the Pareto front. This front, including the pressure field belonging to the maximum *Cl*, maximum *Cl*/*Cd* and minimum *Cd* solutions, is shown in Fig. 2.

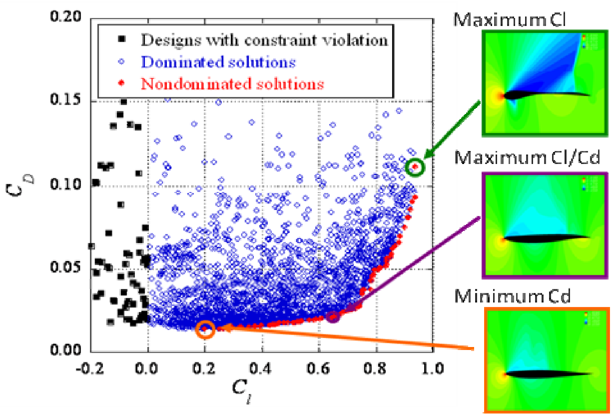

Fig. 2 Properties of all solutions and Pareto front (red)

Based on the parameters of the previous generation new airfoils are selected using combinations and mutations of the current designs. A strong preference is given to the properties of the airfoils that have good Pareto rankings. Moreover different refinements are included in the method such as reintroducing good designs of past generations to accelerate convergence to the true Pareto-front.

The evolutionary algorithm applied in the current research uses 60 generations, each containing 64 individual designs. The conditions for the 2D CFD simulation are shown in Tab. 2. For the equations and mesh size see Tab. 3.

Tab. 2 Conditions for the CFD simulations

Reynolds number	10^6
Mach number	0.8
Angle of attack	2°

Tab. 3 Equations used for the CFD simulations

Governing equations	Reynolds averaged Navier Stokes
Turbulence Model	Baldwin Lomax
Convective term	HLLEW+MUSCL
Viscous term	2nd order central difference
Time Integration	LU+SGS
Mesh size	201 (chordwise) x 49 nodes

The final step is to perform data mining on the generated data to extract useful design knowledge. In this paper Scatter Plot Matrices (SPM) created by R Software [6] are used. The plots in these matrices contain one design parameter on the horizontal axis and one design objective on the vertical axis. Each point in a plot is based on the values belonging to one specific design.

In general, the analysis of only non-dominated solutions results in better correlations between design parameters and objectives, compared with the analysis of all feasible solutions. Therefore only the non-dominated solutions are used as input data for the SPM's. This implies that the information is only valid near the Pareto front. Since these solutions are the most interesting for an actual design this does not degrade the usefulness of the obtained results.

3. Analysis of B-Spline control points

The correlations between the 12 parameters that define the positions of the B-Spline control points and the aerodynamic performance of the non-dominated solutions has been analyzed.

The movement of each free control point is restricted according to the information in Tab. 4 to limit the design space while keeping good control over the airfoil shape.

Tab. 4 Limits to the movement of B-Spline control points

0.66 <x1<0.99	-0.1<y1<0.1
0.33<x2<0.66	-0.1<y2<0.1
0.01<x3<0.33	-0.1<y3<0.1
0.01<x4<0.33	0.0<y4<0.2
0.33<x5<0.66	0.0<y5<0.2
0.66<x6<0.99	0.0<y6<0.2

To assess the performance of each parameter the Pearson correlation coefficient is computed for n designs:

With $x_1 \cdots x_n$ the input data (the values of a single airfoil parameter from all individual designs) and $y_1 \cdots y_n$ the corresponding output

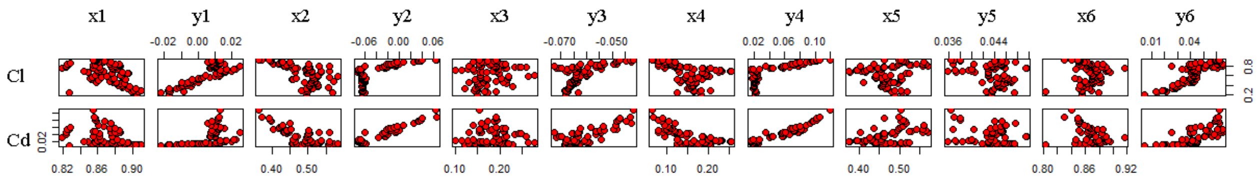

Fig. 3 SPM of parameters and objectives using B-spline parameterization

data (aerodynamic performance coefficients). A bar above a variable indicates the average value of an entire dataset. The correlation coefficients between all combinations of design parameters and objectives are shown in Tab. 5. The solutions with an absolute correlation coefficient ≥ 0.7 are printed in bold to indicate reasonably good correlation. In this case 10 out of 24 correlation coefficients belong to this group.

Tab. 5 Correlation coefficients between B-spline control points and design objectives

	Cl	*Cd*
X1	−0.53	−0.39
Y1	**0.84**	0.51
X2	−0.61	**−0.80**
Y2	**0.72**	**0.95**
X3	0.17	0.08
Y3	0.65	**0.80**
X4	−0.46	**−0.70**
Y4	**0.79**	**0.97**
X5	0.16	0.38
Y5	−0.28	−0.26
X6	0.10	−0.25
Y6	**0.83**	**0.70**

The correlations between parameters and aerodynamic performance are graphically represented in a scatter plot matrix (see Fig. 3).
While the correlation between B-Spline control points and aerodynamic performance is reasonable there is an import disadvantage: the effect of moving the control points on the shape of the airfoil is not straightforward. This makes it difficult to interpret the results of the data-mining process.

4. Analysis of PARSEC parameters

A parameterization method that allows good control over the airfoil geometry is PARSEC. In contrast to B-Spline curves this method is specifically designed for use with airfoils. PARSEC parameters describe the upper and lower surface independent of each other using 6th order polynomials of the following kind:

x [illegible] 0

The advantage of the representation using half powers is that for all derivatives tend to infinity providing perfect continuity at the leading edge. The shape at the trailing edge is C0 continuous which confirms to the usual design of airfoils. In the current research a modified PARSEC method is used with fixed points for the leading and trailing edge for both polynomials.

This results in the following nine parameters:

- RLE: Leading edge radius
- XUP: X-location of the maximum Z point on the upper surface
- XLO: X-location of the minimum Z point on the lower surface
- ZUP: Maximum Z value at the upper surface
- ZLO: Minimum Z value at the lower surface
- ZXXUP: Second derivative at the location of the maximum Z value on the upper surface
- ZXXLO: Second derivative at the location of the minimum Z value on the lower surface
- αTE: Trailing edge angle with the horizontal axis
- βTE: Trailing edge angle between upper and lower surface

For a graphical representation of these parameters see Fig. 5.

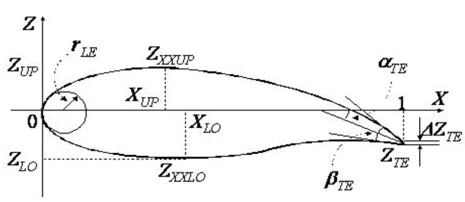

Fig. 5 PARSEC parameters

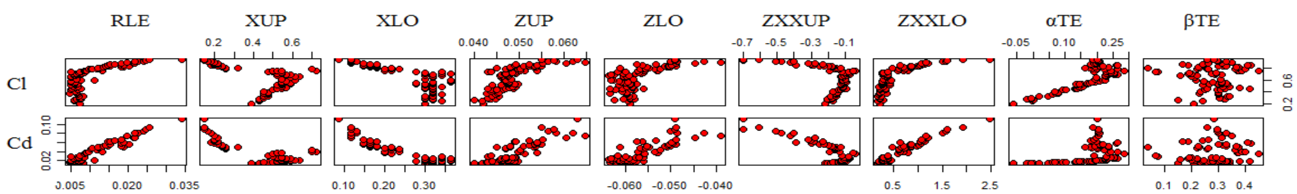

Fig. 4 SPM of parameters and objectives using PARSEC parameterization

The correlation coefficients between the PARSEC parameters and the design objectives are shown in Tab. 6. In this case the PARSEC parameters are obtained from the airfoil shapes of the Pareto-optimal solutions that are optimized with B-Spline curve parameterization.

Tab. 6 Correlation coefficients between PARSEC parameters and design objectives

	Cl	*Cd*
RLE	**0.72**	0.67
XUP	−0.53	**−0.81**
XLO	**−0.77**	**−0.93**
ZUP	**0.76**	**0.84**
ZLO	0.68	**0.83**
ZXXUP	−0.45	**−0.83**
ZXXLO	**0.76**	**0.96**
αTE	**0.82**	0.44
βTE	-0.01	0.01

11 out of 18 coefficients have an absolute value ≥ 0.7. Three parameters provide good correlation with both lift and drag. The scatter plot matrices of the PARSEC parameters and design objectives are available in Fig. 4.

It is remarkable that the parameter βTE shows a far worse correlation compared with all other parameters. This is probably a limitation of the B-spline method and not a limitation of the parameter itself. The current way the boundary conditions are handled at the start- and endpoint of the B-spline curve edge provides very limited control over the shape of the trailing edge. This prevents the evolutionary algorithm from optimizing the shape of the trailing edge independent of the rest of the airfoil geometry.

5. Analysis of the proposed parameters

In general there are many ways to describe the shape of an airfoil, for example:

- Upper and lower surface
- Camber line and thickness distribution
- Upper surface and thickness distribution
- Lower surface and thickness distribution
- Defining the entire contour at once using x=f(j) and z=g(j)

The PARSEC method employs the first option. In addition to this method camber and thickness are often successfully used to explain differences in aerodynamic performance between various airfoil types. Therefore a new parameterization method is proposed that independently describes the camber line and thickness distribution. The camber line has equal distance to the upper and lower surface for any x-value. The following polynomial defines this line:

$$z = \sum_{i=1}^{6} a_i x^i$$

The thickness distribution follows from:

These polynomials guarantee that the same properties regarding continuity at the leading and trailing edge are preserved compared with PARSEC. The leading edge and trailing edge are fixed to respectively (x,z)=(0,0) and (1,0). The following parameters define the camber line:

- ZXLEC: First derivative at leading edge
- Z50C: Z value at 50% of the chord length
- ZX50C: First derivative at 50% of the chord length
- ZXX50C: Second derivative at 50% of the chord length
- ZXTEC: First derivative at the trailing edge

The thickness distribution parameterized using:

- XMAT: X location of the maximum Z value
- ZMAT: Maximum Z value
- ZXXMAT: Second derivative at the maximum Z value
- ZXTET: First derivative at the trailing edge
- ZXXTET: Second derivative at the trailing edge

For a graphical representation of the proposed parameters see Fig. 7.

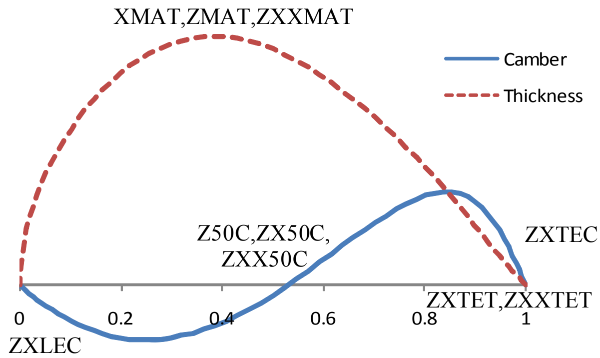

Fig. 7 Proposed parameters

The unknown coefficients in the polynomials can be obtained from the parameters stated above using the following procedure:

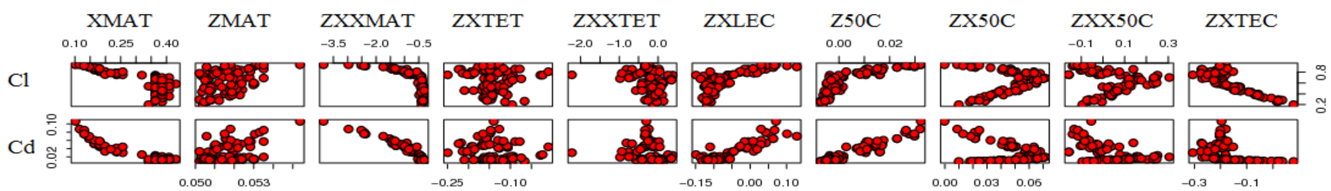

Fig. 6 SPM of parameters and objectives using the proposed parameter set

- Define the polynomials.
- Calculate the first and second derivatives.
- Substitute the x-values belonging to the above parameters in either the original polynomial or one of the derivatives. This results in the left hand side of each equation. The right hand side consists of either the value of the corresponding parameter or the value zero (in case of setting the first derivate to zero at XMAT). Moreover the fixed location of the trailing edge results in two additional equations.
- Finally the system of non-linear equations can be solved for the unknown coefficients a_i .

The correlation coefficients are once more calculated for each combination of parameters and design objectives of the Pareto-optimal solutions optimized with B-Spline curve parameterization. The results are shown in Tab. 7.

Tab. 7 Correlation coefficients between proposed parameters and design objectives

	Cl	*Cd*
XMAT	**−0.79**	**−0.94**
ZMAT	0.35	0.44
ZXXMAT	**−0.70**	**−0.95**
ZXTET	−0.06	−0.11
ZXXTET	−0.24	−0.17
ZXLEC	**0.80**	**0.92**
Z50C	**0.82**	**0.95**
ZX50C	0.14	−0.38
ZXX50C	−0.18	−0.53
ZXTEC	**−0.82**	−0.43

9 out of 20 coefficients have an absolute value ≥ 0.7 . At first sight the performance is slightly worse compared with PARSEC while an advantage is that 4 parameters provide good correlation with both lift and drag at the same time.

However, when observing the scatter plot matrix some of the combinations that have low correlation coefficients actually consist of two rather linear sections (see Fig. 6). This is opposed to the very "random" pattern that is visible for the B-Spline parameters with low correlation coefficients. In this case the correlation coefficient does not correctly represent the quality of the information in the plots. The fact that there seem to be two regions in the design space showing different behavior is valuable design information by itself.

Two correlations that clearly show this phenomenon are the correlation between ZX50C and ZXX50C and *Cl*. To analyze this issue the camber line is plotted in Fig. 8 for different *Cl* values. For low to reasonably high *Cl* values the camber line near the leading edge is very similar. The negative camber line near the leading edge keeps the upper side of the airfoil relatively flat, while the thickness increases when moving in chordwise direction. Solutions without this feature result in very strong shockwaves that greatly increase drag and are therefore not part of the Pareto front. In this range of *Cl* values the increase in lift originates from a stronger camber near the trailing edge, causing higher gradients at 50% chord length when *Cl* increases. Only in case of very high *Cl* values, shockwaves are inevitable and the correlation starts to behave differently. Therefore splitting the design space into solutions with weak or strong shockwaves, would lead to significant improvements of the correlation coefficients between design parameters and aerodynamic performance.

The poor correlation coefficients belonging to ZMAT are caused by the fact that all Pareto-optimal solutions stick to the minimum thickness condition of 10% of the chord length. Increasing this thickness would cause additional drag for a given value of the lift coefficient. This means that the current results are not valid for a less restrictive boundary condition.

The cause of the weak correlation of ZXTET and ZXXTET with aerodynamic performance is suspected to be the same problem that affects the PARSEC parameter βTE: insufficient control over the trailing edge geometry by the current B-spline-curve based optimization.

In conclusion both PARSEC and the proposed parameterization are useful tools for design exploration with their own advantages and disadvantages.

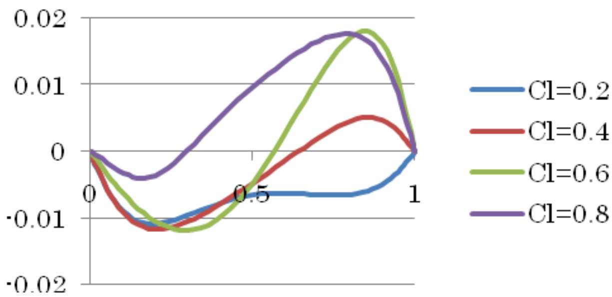

Fig. 8 Camber line of several Pareto optimal solutions

6. Conclusion

A new parameter set to control the airfoil geometry for transonic aerodynamic airfoil shape optimization has been proposed. The correlation of the proposed parameters with aerodynamic performance is similar compared to PARSEC, while more parameters show good correlation with both lift and drag at the same time. This provides insight on the tradeoffs involved with transonic airfoil design. Both PARSEC and the proposed parameterization provide better correlation with the design objectives than B-Spline control points. An additional advantage of both methods is that the parameters have a clear effect on the airfoil geometry. A straightforward relation between parameters and airfoil shape is important to be able to use the results from the data-mining process to increase design knowledge.

It is expected that the performance of some parameters is limited by the solutions obtained by means of B-Spline curve based optimization. Moreover, the ability of each parameterization to allow precise control over the airfoil geometry requires additional attention. Therefore, in the future optimization results of the proposed parameter set and PARSEC parameters will be compared. This will provide more detailed information about both the correlation between parameters and aerodynamic performance, as well as the feasibility of using the proposed parameter set for transonic aerodynamic airfoil shape optimization.

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