A New Efficient and Useful Robust Optimization Approach – Design for Multi-Objective Six Sigma

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Abstract- An efficient and useful robust optimization approach, design for multi-objective six sigma (DFMOSS), has been developed. The DFMOSS couples the ideas of design for six sigma (DFSS) and multi-objective genetic algorithm (MOGA) to solve drawbacks of DFSS. DFMOSS obtains trade-off solutions between optimality and robustness in one optimization. In addition, it does not need careful parameter tuning. Robust optimizations of a test function and welded beam design problem demonstrated that DFMOSS is more effective and more useful than DFSS.

1 Introduction

Design optimization approaches have been applied to various engineering design problems aiming at better design and automated design process. However, because of errors and uncertainties in design process, manufacturing process, and operating condition in real-world engineering designs, a design optimized by a traditional design optimization method seeking only optimality may not achieve its expected performance.

Thus, the idea of robust optimization considering both the optimality and the robustness of objective function and constraints has been paid attention to for real-world design problems in recent years. Here, robustness is defined as stability of system against uncertainties, *e.g.*, the dispersion of performance parameters due to the dispersion of design parameters caused by design errors or uncertainties. Figure 1 illustrates a robust optimization and a traditional optimization. The solution A obtained by a traditional optimization is the best in terms of the optimality but disperses widely in terms of the objective function against the dispersion of the design variable, and this dispersion may extend to the infeasible range. On the other hand, the solution B obtained by a robust optimization is moderately good in terms of the optimality and also good in terms of the robustness; dispersion of the robust solution is narrow against the dispersion of the design variable.

Optimality and robustness of an objective function is usually competing in real-world problems. Therefore, there exist multiple compromised solutions between the optimality and the robustness. Objective of a robust optimization problem is to find these compromised solutions to reveal the trade-off information and to give chance to the upper-level decision maker to select one solution from the compromised solutions with other consideration.

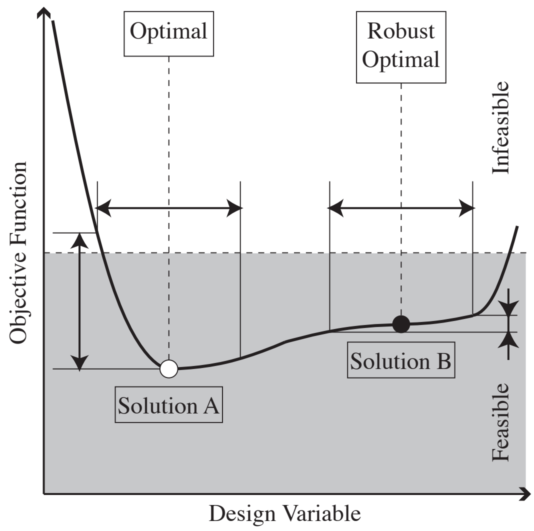

Figure 1: Comparison of traditional optimization and robust optimization

Up to the present, some robust optimization approaches have been developed and investigated to obtain robust optimal solutions[engineous02, huyse02, putko02, youn04, deb05]. Among these, a design for six sigma (DFSS)[engineous02] is one of popular robust optimization approaches because the formulation of DFSS is simpler than that of other approaches. In fact, DFSS has been successfully applied to various robust optimization problems in various engineering fields[koch02, shimoyama04]. However, this approach has some drawbacks. First, DFSS finds only one robust optimal solution in one optimization. Second, DFSS does not guarantee that the obtained robust optimal solution satisfies the specified sigma level. Therefore, users need to repeat robust optimization many times by using DFSS with different input parameter set until a satisfactory robust optimal solution is obtained. In this sense, DFSS is not efficient and not useful.

Multi-objective genetic algorithm (MOGA)[deb01] is an evolution-based design optimization algorithm for multi-objective optimization problems. Because the MOGA deals with multiple solutions simultaneously and evaluates these solutions based on Pareto-optimality concept, MOGA finds multiple optimal solutions of multi-objective optimization problem in one optimization and obtains the trade-off relation between competing objectives effectively.

Therefore, objective of the present study is to develop a new efficient and useful robust optimization approach by combining the ideas of DFSS and MOGA. To ensure efficiency of the current approach named as “design for multi-objective six sigma (DFMOSS),” robust optimization of a test function and the welded beam design problem are demonstrated. Rest of the paper is organized as follows: Chapter 2 briefly describes DFSS. Chapter 3 presents idea of DFMOSS. Chapter 4 presents simulation results. Chapter 5 concludes current study.

2 Design for Six Sigma

Design for six sigma (DFSS)[engineous02] is one of conventional robust optimization approaches. Here, the term “sigma” refers to standard deviation σ, which is a measure of dispersion, and “six sigma” is one of the management reform techniques aiming at the establishment of business process with very small dispersion such that 6σ is included in the acceptable performance range. The level of dispersion can be defined as “sigma level n”, as shown in Fig. 2. Larger sigma level indicates smaller dispersion, i.e., more robust characteristic. Here, Fig. 2 shows the normal distribution case but “six sigma” concept can be applied to any probability density distribution cases.

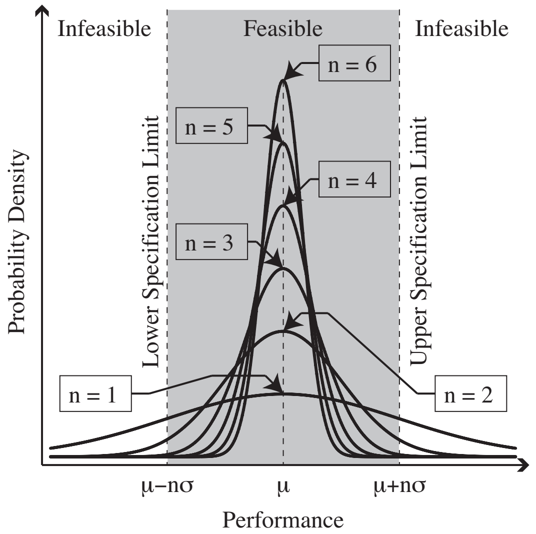

Figure 2: Characteristics of sigma level n

In a normal optimization (minimization) problem, objective function f of design variable $\boldsymbol{x}$ should be minimized as follows.

$$\text{Minimize: } f(\boldsymbol{x}) \tag{1}$$

In DFSS, this normal optimization problem is rewritten to the problem in which both the mean value μ_f and the variance ${\sigma_f}^2$ of objective function $f(\boldsymbol{x})$ should be minimized as follows.

$$\text{Minimize: } w_\mu \mu_f + w_\sigma {\sigma_f}^2 \tag{2}$$

where w_μ and w_σ are weighting factors. In addition, following inequalities should be satisfied as constraints on sigma level.

$$\begin{aligned} \text{Subject to: } \mu_f - n\sigma_f &\geq \text{LSL} \\ \mu_f + n\sigma_f &\leq \text{USL} \end{aligned} \tag{3}$$

Here, n denotes the sigma level and LSL/USL denote the lower/upper specification limits, respectively.

Figure 3 illustrates flowchart of robust optimization using DFSS. First, parameters such as weighting factors w_μ and w_σ, sigma level n, and LSL/USL are specified by users, and then it proceeds to the optimization block. In this block, μ_f and σ_f of $f(\boldsymbol{x})$ at sample points around $\boldsymbol{x}$ are evaluated, and $w_\mu \mu_f + w_\sigma {\sigma_f}^2$ is treated as one objective function. Then, $\mu_f - n\sigma_f - \text{LSL}(\geq 0)$ and $\mu_f + n\sigma_f - \text{USL}(\leq 0)$ are evaluated as two constraint functions. $\boldsymbol{x}$ in next step is reproduced based on the evaluated objective and constraint functions, and this optimization process is iterated until $\boldsymbol{x}$ is converged. This single-objective optimization can be carried out by using any single-objective optimization approaches.

DFSS illustrated in Fig. 3, however, has some drawbacks as follows.

- **It is necessary to set weighting factors w_μ and w_σ carefully in advance.** There exists arbitrariness in specification of the weighting factors in Eq. 2. Users need to specify value of these parameters according to balance between the optimality and the robustness they expect. However, it is difficult for users to specify value of weighting factors appropriately in advance because the trade-off information is unknown. Eventually, users can not help carrying out robust optimization many times with different weighting factor values until satisfactory solution is obtained.
- **It is necessary to set appropriate sigma level n in advance.** Essentially, the sigma level satisfying Eq. 3 is known only after an optimization. However, users must specify the sigma level blindly without any information. If users specify the sigma level too large, a robust optimal solution may not be obtained by the optimization because the constraints for sigma level become too severe. On the other hand, if users set the sigma level too small, the robust optimal solution obtained by the optimization may have lack of reliability. Eventually, users need to carry out robust optimization repeatedly until a feasible robust optimal solution is found.
- **Only one robust optimal solution is obtained in one optimization.** Because DFSS deals with single-objective optimization problem considering the weighted summation of the mean value and the variance of objective function ($w_\mu \mu_f + w_\sigma {\sigma_f}^2$) as a new objective function, only one robust optimal solution is obtained by one DFSS optimization. Therefore, users must carry out many optimizations with different weighting factor values or the sigma level to obtain multiple robust optimal solutions. Moreover, many optimizations can not necessarily derive

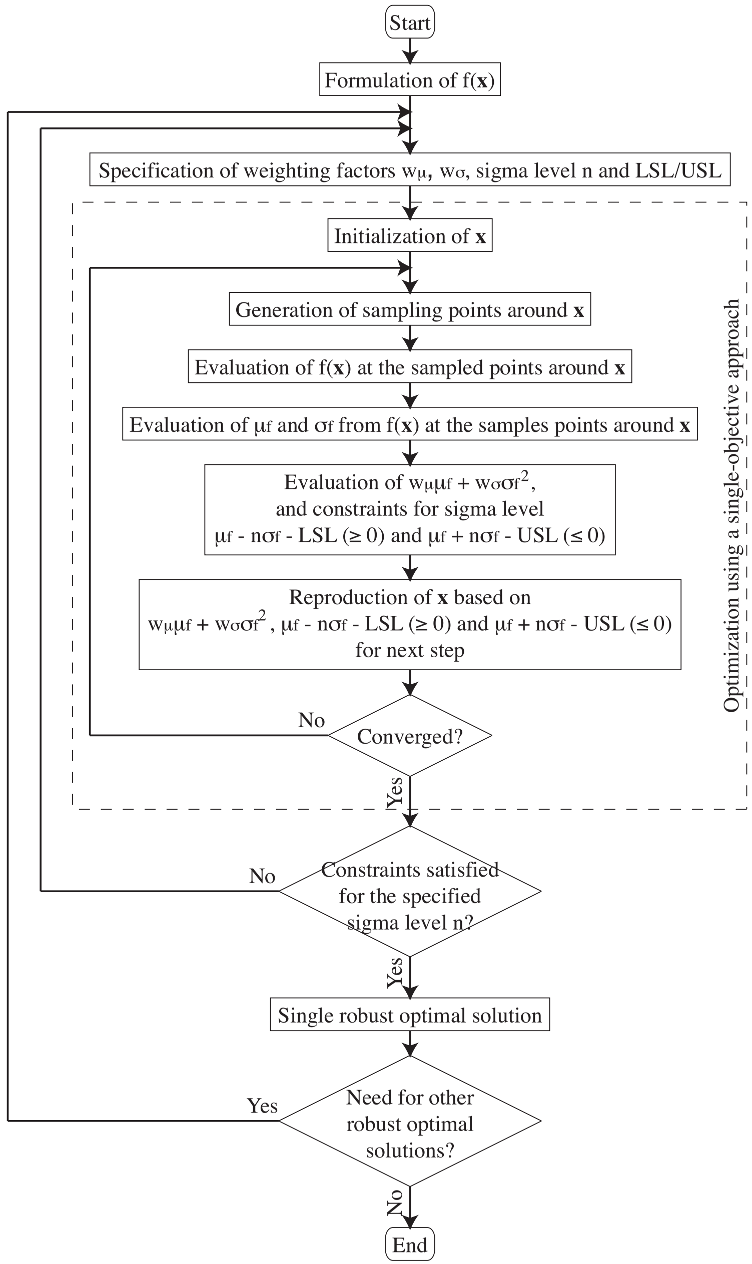

Figure 3: Flowchart of robust optimization using DFSS

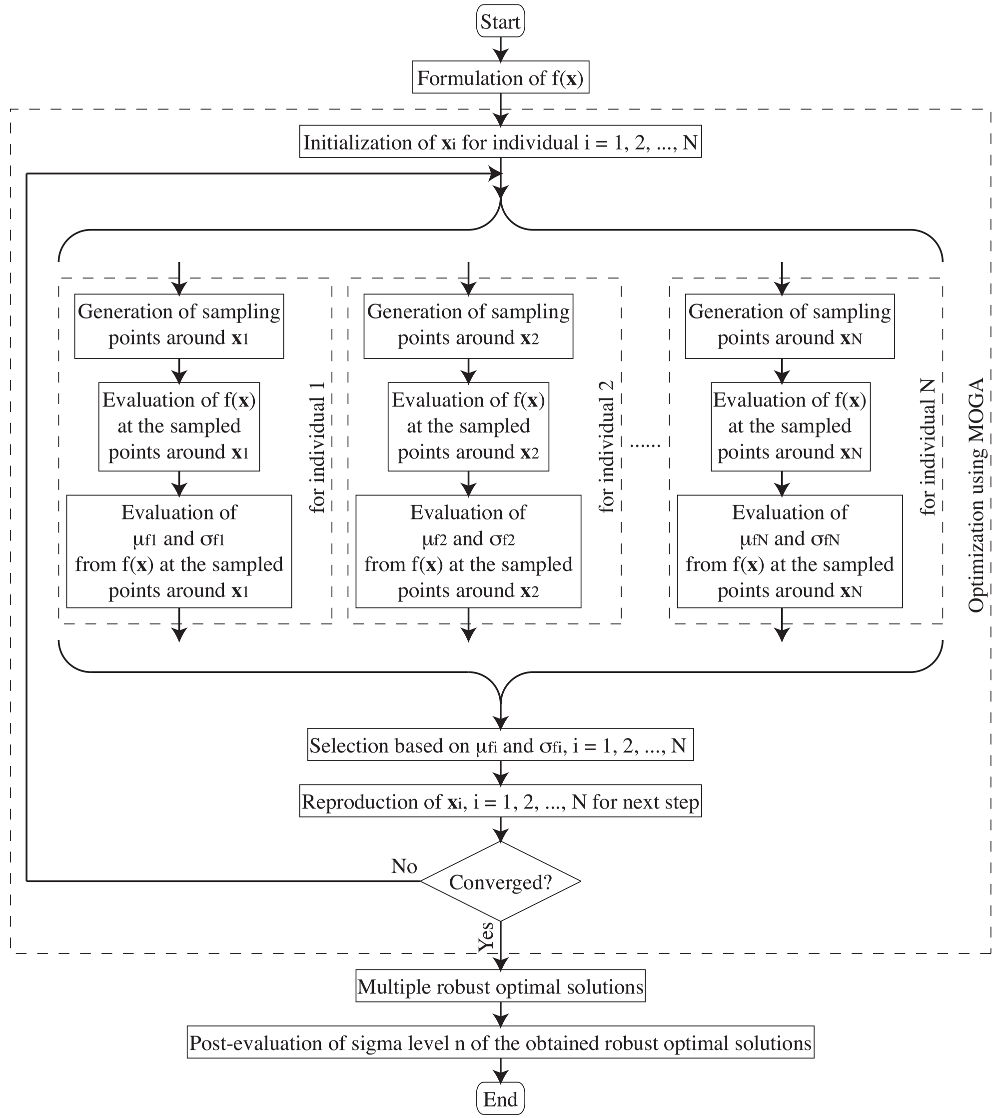

Figure 4: Flowchart of robust optimization using DFMOSS

trade-off relation between the optimality and the robustness.

3 Design for Multi-Objective Six Sigma

Drawbacks of the DFSS, as explained in the previous subsection, mainly come from the fact that the DFSS deals with the single-objective optimization problem as shown in Eq. 2. Therefore, idea of design for multi-objective six sigma (DFMOSS) developed by the authors is to incorporate multi-objective genetic algorithm (MOGA)[deb01] into DFSS to solve the drawbacks of the DFSS. In DFMOSS, the mean value μ_f and the standard deviation σ_f of objective function $f(\boldsymbol{x})$ are treated as new multiple objective functions and minimized separately as follows.

$$\begin{aligned} \text{Minimize: } & \mu_f \\ & \sigma_f \end{aligned} \tag{4}$$

Because the formulation of DFMOSS does not include the weighting factors w_μ and w_σ which are seen in the formulation of DFSS (Eq. 2), DFMOSS does not have difficulty in the advance specification of weighting factors. In addition, solving this multi-objective optimization problem by using MOGA results in many robust optimal solutions in one optimization. Furthermore, DFMOSS does not consider the constraints for sigma level which are seen in the formulation of DFSS (Eq. 3) during optimization process. Thus, DFMOSS also does not have difficulty in the advance specification of sigma level n. The sigma level satisfying Eq. 3 can be evaluated from the robust optimal solutions obtained by the optimization as post-processing, as explained later.

Figure 4 illustrates flowchart of robust optimization using DFMOSS. There is no need to specify parameters such as weighting factors w_μ and w_σ, sigma level n and LSL/USL before optimization block, which is seen in the DFSS (Fig. 3). In the optimization block using MOGA, multiple solutions (individuals) $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N$ are dealt with simultaneously. For each individual $i = 1, 2, \cdots, N$, μ_{f_i} and σ_{f_i} are evaluated as two separate objective functions from $f(\boldsymbol{x})$ at the sampled points around $\boldsymbol{x}_i$. Better solutions are selected based on the Pareto-optimality concept between μ_{f_i} and σ_{f_i} for $i = 1, 2, \cdots, N$. Solutions $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N$ in next step are reproduced by crossover and mutation from the selected solutions. This optimization process is iterated until the trade-off relation between μ_f and σ_f is converged, and multiple robust optimal solutions are obtained.

In DFMOSS, the sigma level n satisfying Eq. 3 is evaluated from the obtained robust optimal solutions after the optimization block, as illustrated in Fig. 4. Figure 5 illustrates detail of post-evaluation of sigma level. Now it is assumed that four robust optimal solutions (solution A, B, C and D) are obtained by a DFMOSS optimization. Painted area indicates the area satisfying Eq. 3. Figure 5 shows that the solution C is included in the painted area, that is, this solution has more than $n\sigma$ robustness quality. If the sigma level n becomes smaller, the painted area becomes larger (the gradient of broken lines by which the painted area is bound becomes steeper) and the other solutions A, B and D may be included in the painted area. Therefore, Fig. 5 also shows that the other solutions A, B and D may have smaller sigma level, i.e., less robust characteristic than the solution C.

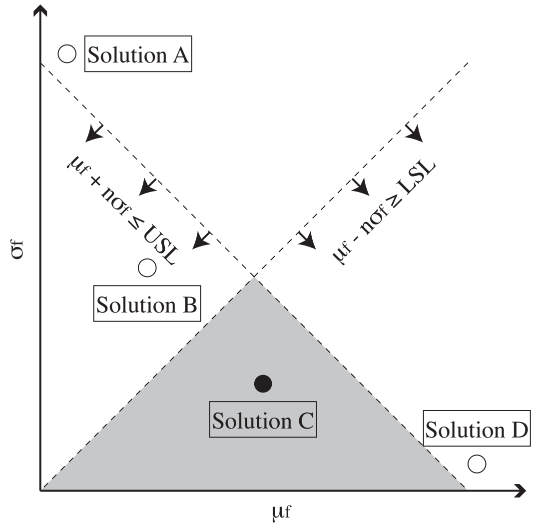

Figure 5: Post-evaluation of sigma level

4 Simulation Results

The DFMOSS is applied to two robust optimization problems, and the simulation results obtained by using the DFMOSS are compared to those obtained by using the DFSS. The first is a test function optimization problem, and the second is a welded beam design problem.

4.1 Test Function Optimization Problem

A test function $f(x)$ is defined as follows.

$$f(x) = -\exp\left(-\frac{|x|}{5}\right)\cos\left(\frac{2\pi x}{|x|^{0.1}}\right) \tag{5}$$

where the range of design variable is $-0.5 \leq x \leq 5$. Now the robust optimization problem that both the mean value μ_f and standard deviation σ_f of objective function $f(x)$ should be minimized is considered.

Figure 6 shows the test function distribution ($f(x)$ against x). In a normal optimization problem that the objective function $f(x)$ should be minimized, it is clear that an optimal solution is $x = 0$. In the robust optimization problem, on the other hand, $x = 0$ is the best in terms of the optimality, but not good in terms of the robustness because the objective function $f(x)$ disperse widely against the dispersion of design variable x around $x = 0$. The optimality becomes worse but the robustness becomes better in order of hollows $x = 0$, 1, 2.16, 3.38 and 4.66 because the hollow becomes shallower and gentler. Therefore, all these five hollows can be the robust optimal solutions in this robust optimization problem.

Table 1: Optimization conditions of test function optimization problem

for SOGA/MOGA	Population size	32
	Number of generations	100
	Mutation rate [%]	20
for MCS random design variable	Sample size	1000
	Probability density distribution	normal distribution
	Mean value	value of each solution
	Standard deviation	0.1
for DFSS	USL	-0.2
	LSL	N/A
	Sigma level	3σ
	$w_\mu : w_\sigma$	1:1000, 1:100, 1:10 1:1, 10:1, 100:1 1000:1

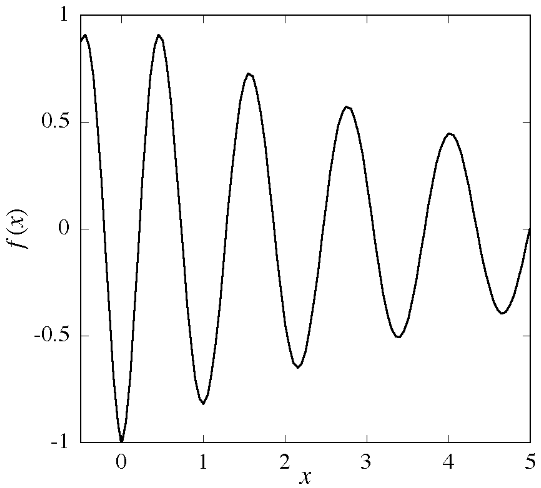

Figure 6: Test function distribution

This robust optimization problem is solved by using the MOGA coupled with the DFMOSS, and the single-objective GA (SOGA) coupled with the DFSS, respectively. In both robust optimizations, the blended crossover (BLX-0.5), random mutation and the Best-N selection are used. Selection and constraint-handling are based on the Pareto-based constraint-handling (PBCH) approach[oyama05] using the Pareto-ranking method[fonseca93] and the fitness sharing[fonseca93]. The statistical values of objective function (μ_f and σ_f) are evaluated by the Monte Carlo simulation (MCS) with descriptive sampling (DS)[saliby90].

Table 1 shows the optimization conditions of test function optimization problem. The mean value of MCS random design variable is set as the design variable of each solution. The standard deviation of MCS random design variable, on the other hand, is fixed at 0.1. In the robust optimizations using DFSS, the sigma level is set as 3σ in advance, and total seven cases with different combination of weighting factors ($w_\mu : w_\sigma$) are carried out. In the robust optimization using DFMOSS, on the other hand, it is not necessary to set the sigma level in advance, and only one case is carried out without setting weighting factors, as explained in the previous section.

Figure 7 shows the robust optimal solutions (σ_f against μ_f) and Table 2 shows the numerical data of the robust optimal solutions of the test function optimization problem obtained by using the DFSS and the DFMOSS, respectively. In the robust optimization using DFSS, only two robust optimal solutions with more than 3σ robustness quality ($x = 1$ and 3.39) are obtained though total seven-time optimizations with different combination of weighting factors are carried out. Here it is noted that essentially there exist total three robust optimal solutions with more than 3σ robustness quality ($x = 1$, 2.16 and 3.38). These results indicate that the DFSS does not have enough capability of finding all robust optimal solutions. In this study, two robust optimal solutions can be obtained because the advance setting of sigma level as 3σ is appropriate by chance. However, it is not always guaranteed for the DFSS to obtain the robust optimal solutions according to the advance setting of sigma level.

In the robust optimization using DFMOSS, on the other hand, all five robust optimal solutions ($x = 0$, 1, 2.16, 3.38 and 4.66) can be obtained successfully and effectively in only one calculation, and it is understood easily that the maximum sigma level which the robust optimal solutions have is more than 3σ by the post-evaluation.

4.2 Welded Beam Design Problem [deb91, mezura-montes03]

The welded beam structure is shown in Fig. 8. The welded beam consists of a beam and a weld required to secure the beam to the member. The objective of the design is to find a feasible set of dimensions h, l, t and b (denoted by $\boldsymbol{x} = [x_1, x_2, x_3, x_4]$) to carry a certain load (P) and still have a minimum total fabricating cost.

The objective function $f(\boldsymbol{x})$ is the total fabricating cost which mainly comprises of the set-up cost, welding labor cost and material cost:

$$f(\boldsymbol{x}) = (1 + c_1)\, {x_1}^2 x_2 + c_2 x_3 x_4\, (L + x_2) \qquad (6)$$

where c_1 and c_2 are the cost of unit volume of weld material and bar stock, respectively. The associated functional

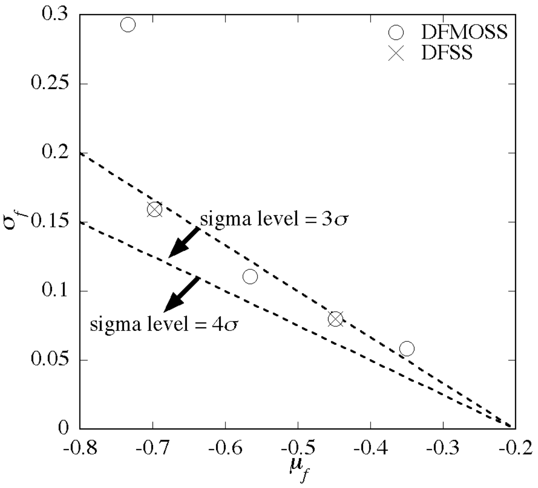

Figure 7: Robust optimal solutions and sigma level of test function optimization problem

Table 2: Numerical data of robust optimal solutions of test function optimization problem

	$w_\mu : w_\sigma$	x	μ_f	σ_f
DFSS	1:1000	3.3864	-0.44811	0.079737
	1:100	3.3863	-0.44812	0.079737
	1:10	0.99987	-0.69769	0.1592
	1:1	0.99808	-0.69776	0.15927
	10:1	0.99724	-0.69777	0.15934
	100:1	0.99713	-0.69777	0.15935
	1000:1	0.99712	-0.69777	0.15935
DFMOSS		0.001737	-0.73406	0.29317
		1.0023	-0.69747	0.15923
		2.159	-0.56617	0.11038
		3.3835	-0.44815	0.079798
		4.6596	-0.3499	0.058399

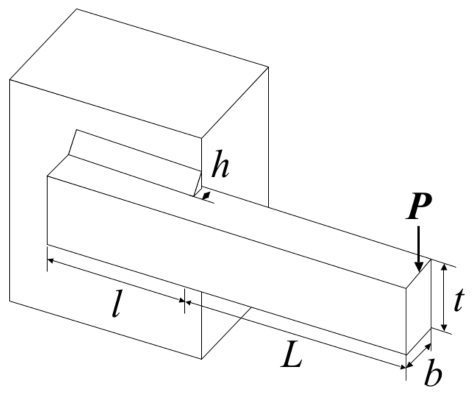

Figure 8: Welded beam structure

constraints are:

$$
\begin{aligned}
\text{Subject to: } & \tau(\boldsymbol{x}) - \tau_{\max} \leq 0 \\
& \sigma(\boldsymbol{x}) - \sigma_{\max} \leq 0 \\
& x_1 - x_4 \leq 0 \\
& c_1 {x_1}^2 + c_2 x_3 x_4 \left(L + x_2\right) - 5 \leq 0 \\
& \delta(\boldsymbol{x}) - \delta_{\max} \leq 0 \\
& P - P_c(\boldsymbol{x}) \leq 0
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
\tau(\boldsymbol{x}) &= \sqrt{\tau'^2 + 2\tau'\tau''\frac{x_2}{2R} + \tau''^2} \\
\tau' &= \frac{P}{\sqrt{2}x_1x_2}, \quad \tau'' = \frac{MR}{J} \\
M &= P\left(L + \frac{x_2}{2}\right) \\
R &= \sqrt{\frac{{x_2}^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \\
J &= 2\left\{\sqrt{2}x_1x_2\left[\frac{{x_2}^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \\
\sigma(\boldsymbol{x}) &= \frac{6PL}{x_4{x_3}^2}, \quad \delta(\boldsymbol{x}) = \frac{4PL^3}{E{x_3}^3x_4} \\
P_c(\boldsymbol{x}) &= \frac{4.013E\sqrt{{x_3}^2{x_4}^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
c_1 &= 0.10471, \quad c_2 = 0.04811 \\
P &= 6 \times 10^3[\text{lb}], \quad L = 14[\text{in}] \\
E &= 3 \times 10^7[\text{psi}], \quad G = 1.2 \times 10^7[\text{psi}] \\
\delta_{\max} &= 0.25[\text{in}], \quad \tau_{\max} = 1.36 \times 10^4[\text{psi}] \\
\sigma_{\max} &= 3 \times 10^4[\text{psi}]
\end{aligned}
\tag{9}
$$

where $\tau(\boldsymbol{x})$, $\sigma(\boldsymbol{x})$, $\delta(\boldsymbol{x})$ and $P_c(\boldsymbol{x})$ are weld shear stress, bar bending stress, bar end deflection and bar buckling load, respectively. The ranges of design variables are $0.125 \leq x_1 \leq 5$[in], $0.1 \leq x_2 \leq 10$[in], $0.1 \leq x_3 \leq 10$[in] and $0.1 \leq x_4 \leq 5$[in]. Now the robust optimization problem that both the mean value μ_f and the standard deviation σ_f of total fabricating cost $f(\boldsymbol{x})$ should be minimized is considered.

The optimization methods are the same as those used in the test function optimization problem in the previous subsection. Table 3 shows the optimization conditions of welded beam design problem. In the robust optimization using DFSS, the sigma level is set as 6σ in advance, and total seven cases with different combinations of weighting factors ($w_\mu : w_\sigma$) are carried out. In the robust optimization using DFMOSS, on the other hand, it is not necessary to set the sigma level in advance, and only one case is carried out without setting weighting factors.

Figure 9 shows the robust optimal solutions (standard deviation against mean value of total fabricating cost) of welded beam design problem. In the robust optimization

Table 3: Optimization conditions of welded beam design problem

for SOGA/MOGA	Population size	50
	Number of generations	1000
	Mutation rate [%]	20
for MCS random design variables	Sample size	1000
	Probability density distribution	normal distribution
	Mean value	value of each solution
	Standard deviation	0.01
for DFSS	USL	3
	LSL	N/A
	Sigma level	6σ
	$w_\mu : w_\sigma$	1:1000, 1:100, 1:10, 1:1, 10:1, 100:1, 1000:1

using DFSS, robust optimal solutions with more than 6σ robustness quality are obtained. However, these solutions distribute very locally though total seven-time optimizations with different combination of weighting factors are carried out. These results indicate that the DFSS does not have enough capability of finding robust optimal solutions globally. In this study, the robust optimal solutions can be obtained because the advance setting of sigma level as 6σ is appropriate by chance. However, it is noted that it is not always guaranteed for the DFSS to obtain the robust optimal solutions according to the advance setting of sigma level.

In the robust optimization using DFMOSS, on the other hand, many robust optimal solutions can be obtained effectively in only one optimization, and these solutions distribute globally and uniformly. Also it is understood easily that the maximum sigma level which the robust optimal solutions have is more than 6σ and total twenty-one robust optimal solutions have more than 6σ robustness quality by the post-evaluation.

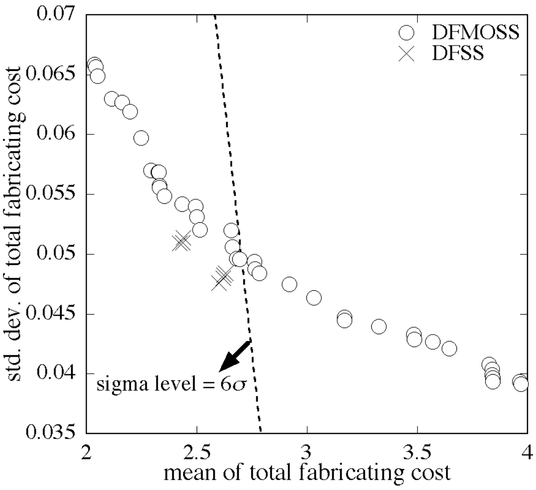

Figure 9: Robust optimal solutions and sigma level of welded beam design problem

5 Conclusions

In this study, a new robust optimization approach DFMOSS was developed by coupling the ideas of DFSS and MOGA. The DFMOSS has some advantages compared to the DFSS as follows.

- It is not necessary to set weighting factors in advance.
- It is not necessary to set the sigma level in advance.
- Multiple robust optimal solution can be obtained in only one optimization.

DFMOSS was applied to test function optimization problem and welded beam design problem. The optimization results obtained by using the DFMOSS were compared to those obtained by using the DFSS. These results showed that the DFMOSS had more effective and more useful characteristics than the DFSS.

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