Coding by Taguchi Method for Evolutionary Algorithms Applied to Aerodynamic Optimization

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Abstract. A new coding technique using Taguchi method is proposed for Evolutionary Algorithm (EA) applied to an aerodynamic optimization. Taguchi method is used to investigate interactions of design variables and to determine the appropriate coding structure for EA in advance. EA coupled with the new coding technique is then applied to aerodynamic design of a transonic wing. Three-dimensional Navier-Stokes calculation is used for estimation of wing performance.

1 INTRODUCTION

An aerodynamic shape optimization problem is a typical hard-to-optimize problem. Because of nonlinearity of the flow equations, aerodynamic objective functions are often rough, discontinuous and multi-modal. To find the global optimum for such complex functions, a robust optimization algorithm is required.

As a robust optimization tool, EA was successfully applied to a subsonic wing optimization using three-dimensional Navier-Stokes (N-S) calculations in [1]. Key features of the method are simplification of airfoil definition according to subsonic wing aerodynamics and parallelization of N-S calculations on Numerical Wind Tunnel (NWT). NWT is a parallel vector machine of peak performance at 279 GFLOPS with 166 processing elements. (NWT was used by winners of IEEE's 1995 & 1996 Gordon Bell Prize for performance.) The resultant design was consistent with the design principles obtained from existing theories and experiments and therefore indicated that EA had found the global optimum.

An extension to a transonic wing optimization, however, is not straightforward. To obtain a good transonic airfoil shape such as supercritical airfoils, airfoil definition with a large degree of freedom is necessary. Such definition requires a large number of design variables and often contains "interactions" of design variables as nonlinearities in fitness functions. These interactions are often referred as "epistasis", corresponding to the term epistasis in biology. Since the optimization process in EA depends on construction of similarity templates (schemata) of design variables, EA cannot find clues to the optimum if epistasis is too large, i.e., small-sized schemata do not exist. A degree of epistasis is strongly related to the performance of EA through sizes of "good" schemata. If episitasis of design variables can be identified in advance, the size of schemata may be reduced by permuting design variables accordingly. Therefore, epistasis analysis provides important information for coding techniques of EA. However, an exhaustive search of epistasis requires as many evaluations as those required for EA itself. Obviously, computational effort for such preprocessing is prohibitive.

In the late 1950s Genichi Taguchi dispersed a statistical tool for quality improvement i.e., a factorial design using orthogonal arrays[3](OA)[2] . It has been developed to gain sufficient information

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[3] Although OA is originally discovered by R.A. Fisher, a factorial design using OA is also called Taguchi method since Taguchi simplified the use of OA by providing tabulated OA and corresponding linear graphs.

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from a structured set of coherent tests at the least expenditure of resources. This technique will be applied to examine the epistasis of design variables here.

In this paper, a tree structure of design variables inspired by the data structure of Genetic Programming is introduced as a coding structure. The coding structure is determined by the epistasis analysis using Taguchi method in advance. Then, the resulting coding structure is applied to aerodynamic design using multiobjective EA[3] coupled with FLO-27[4] to examine the new coding technique. Finally a transonic wing design using three-dimensional N-S calculations will be demonstrated.

2 APPROACH

2.1 GEOMETRY REPRESENTATION OF A WING

In this study, airfoil sections are generated by the extended Joukowski transformation[5]. It transforms a unit circle to various kinds of airfoils in the complex number plane by two consecutive conformal mappings as,

$$Z_0 = re^{i\theta} + Z_c \tag{1}$$

$$Z_1 = Z_0 - \epsilon/(Z_0 - \Delta) \tag{2}$$

$$Z = Z_1 + 1/Z_1 \tag{3}$$

here Z_c, Z_0, Z_1, Z and ϵ are complex numbers and Δ, r, and θ are real numbers, where r is determined so that Z_0 passes the origin of the coordinate axes. This transformation is therefore defined by Z_c, ϵ, and Δ. The present design variables are given by five parameters (x_c, y_c, x_t, y_t, Δ) where a position (x_c, y_c) corresponds to the center of the original circle Z_0, the complex number ϵ corresponds to (x_t, y_t) and Δ is the preliminary movement in the real axis. It is known that x_c, x_t, and Δ are related to the airfoil thickness while y_c and y_t are related to the airfoil camber line.

Then, the wing geometry is represented by these airfoil parameters and twist angles distributed in the spanwise direction. These parameters are interpolated by second-order B-Spline curves. The B-Splines are defined by seven points for the inviscid case and by five points for the viscous case. Planform of the wing is taken from the NASA Energy Efficient Transport (EET) Program[6].

2.2 EVALUATION

2.2.1 AERODYNAMIC PERFORMANCE

In this study, FLO-27 and a N-S solver are used to evaluate aerodynamic performance for inviscid and viscous cases, respectively. Freestream Mach number is 0.8 for both cases and Reynolds number based on the root chord is set to 10^7 for the viscid case.

FLO-27 is applied to epistasis analysis using Taguchi method as well as the inviscid wing optimization. It is a transonic, conservative, full-potential code developed by Jameson and Caughey. The N-S solver used here is based on a TVD-type upwind differencing[7], the LU-SGS scheme[8] and the multigrid method[9]. N-S computations are distributed to 64 processing elements of NWT so that the aerodynamic evaluation processed in parallel.

2.2.2 STRUCTURAL CONSTRAINT

A structural constraint is introduced to the viscous case to obtain a realistic wing in the transonic regime. For the brevity, the wing and its spanwise lift distribution are replaced by a cantilever and concentrated loads, respectively. From the loads, the bending moment distribution is calculated, which gives the structural stress on the wing. Then the constraint is given so that the local stress is less than the ultimate shear stress of Aluminum alloy (see, for example, [10]).

2.3 EVOLUTIONARY ALGORITHM

In EA, design variables of each candidate are encoded into finite-length strings just as the characteristics of an individual are encoded in chromosomes as genes. Here, randomly created design candidates make up the initial population to be optimized according to objective function value (fitness) through the simulated evolution. Evolutionary operators for the evolution process are evaluation, selection, crossover and mutation[11] as shown in Fig. 1. The evolutionary direction operator[13] is a special technique to improve local search performance of EA. In this study, a population size of 64 is used.

2.3.1 *EVOLUTIONARY OPERATORS FOR SINGLE-OBJECTIVE EA*

In EA, selection is a process in which individual strings are copied in mating pool according to their fitness. The ranking method coupled with the Stochastic Universal sampling (SUS)[12] is used. The best and the second best individuals in each generation are copied to the new generation automatically as the elitist strategy.

The crossover operator exchanges chromosomes of the selected parents at random. Here, each spanwise distribution of Joukowski transformation parameters and twist angle is encoded as a chromosome. The simple one-point crossover and the evolutionary direction operator are used to create 70% and 30% of child generation, respectively.

Mutation is a random walk of a string that will occur during the crossover process at a given mutation rate. This operator keeps diversity of a population. Here, mutation takes place at a probability of 10% and then adds a random disturbance to the parameter in the amount up to $\pm$ 10% of design space.

2.3.2 *EVOLUTIONARY OPERATORS FOR MULTIOBJECTIVE EA*

By maintaining a population of solutions, EA can search for many Pareto-optimal solutions in parallel. This characteristic makes EA very attractive for solving multiobjective (MO) problems. The following two features are desired to solve MO problems successfully: 1) The solutions obtained are Pareto-optimal, 2) They are uniformly sampled from the Pareto-optimal set. To achieve these with EA, the ranking selection method and the fitness sharing technique[14] are used. As the elitism, the best-N selection[15] is incorporated, where the best N individuals are selected for the next generation among N parents and N children so that Pareto solutions will be kept once they are formed. Since the strong elitism is used, high mutation rate of 0.5 is applied while the amount of disturbances is reduced from 40% to 1% of the design space as the generation advances.

2.4 TAGUCHI METHOD

A parametric study is often conducted by varying one parameter at a time or by trial and error for a limited number of parameters. However, such approaches only lead to incomplete knowledge for a large design space. An exhaustive search, in contrast, requires unacceptably large number of experiments and thus they are not suitable to real-world problems. For instance, a complete study of a design space of 8 parameters with 3 levels requires 6541 experiments.

Taguchi method is an efficient approach for parametric studies of large systems, based on the statistical theory for design of experiments. It reduces the required number of experiments without deteriorating quality of information by arranging the experiments according to the orthogonal array.

Taguchi method starts with selection of factors and their levels. The number of factors and their levels give the total degree of freedom of experiments, which determines the size of required experiments. Then the orthogonal table is selected according to the size of the experiments. The set of experiments is performed according to this table so that all combinations of the factors become orthogonal.

Then, the effectiveness of factors and their interactions can be estimated statistically by the values of F, the variance of factors and interactions divided by that of error. If F is greater than the critical value, say, 5%, the corresponding factor or interaction is considered effective. In this paper, this method is applied to analyze the epistasis of the Joukowski transformation parameters.

3 RESULTS

3.1 CODING BY TAGUCHI METHOD

Taguchi method is employed to evaluate effectiveness of spanwise variation of design variables on aerodynamic performance C_L and C_D of a wing. Only to account for positive changes in aerodynamic performance (increase of C_L and decrease of C_D), following two functions are introduced:

$$f1 = max(C_L - C_L0, 0) \tag{4}$$

$$f2 = -min(C_D - C_D0, 0) \tag{5}$$

where C_L0 and C_D0 are those of a wing having a constant airfoil section along the spanwise direction. This wing was obtained by optimizing C_L and C_D without spanwise variation of airfoil sections using FLO-27. Airfoil design variables and a twist angle x_c, y_c, x_t, y_t, Δ, α, are considered as factors. Three types of spanwise variations of the factors are considered as levels: no variation, linear increase from root to tip, and vice versa. The number of interactions considered here is ten, i.e., $x_c y_c$, $x_c x_t$, $x_c y_t$, $x_c\Delta$, $y_c x_t$, $y_c y_t$, $y_c\Delta$, $x_t y_t$, $x_t\Delta$ and $y_t\Delta$. Although the exhaustive search of interactions among six factors requires 729 experiments, Taguchi method requires only 81 experiments.

Figure 2 shows F values for the factors and interactions. The solid line and the broken line are critical F values with 1% or 5% statistical risks, respectively. F values more than these critical values are judged effective. While every factor is effective on both f1 and f2, interactions of $x_c x_t$ and $y_c y_t$ appear effective. This result is consistent with the fact that x_c, x_t, and Δ are related to the airfoil thickness while y_c and y_t are related to the airfoil camber line.

The epistasis analysis using Taguchi method indicates strong interactions in (x_c, x_t) and (y_c, y_t). Therefore, a tree structure of design variables can be constructed as shown in Fig. 3. This will be better than the simple-minded sequential coding shown in Fig. 4. When the tree structure coding is used, good schemata, (x_c, x_t) and (y_c, y_t) will be formed easily. The difference between these coding techniques appears in crossover.

3.2 MULTIOBJECTIVE INVISCID OPTIMIZATION

Multiobjective aerodynamic design using EA is first demonstrated for the validity of the new coding technique. The main multiobjective optimization is defined as,

Maximize C_L

Minimize C_D

Here the structural constraint is ignored and the optimization is stopped after 100 generations. Figure 5 shows the Pareto-optimums indicating the tradeoff between maximization of C_L and minimization of C_D. Solid and hollow points show the resulting Pareto fronts obtained from the tree-structure and the sequential coding techniques, respectively. This figure indicates that the present EA with the tree-structure coding outperforms the conventional EA with the sequential coding. Airfoil sections of the designed wings picked from the Pareto fronts where $C_L = 1.5$ are shown in Figs. 6 and 7 for a comparison purpose. The design obtained from the new coding technique has a large leading-edge radius, reduced curvature over the middle region of the upper surface and substantial aft chamber. This indicates that the new coding technique helps finding supercritical airfoils suitable to transonic flows while the sequential coding fails.

3.3 SINGLE-OBJECTIVE VISCOUS OPTIMIZATION

The present EA is applied to a wing optimization using N-S calculations. Evaluations are parallelized to reduce the computational time necessary for evaluations using a N-S solver. The objective is lift-to-drag ratio to be maximized with a penalty for the structural constraint. This problem is the same as [16].

The convergence history is illustrated in Fig. 8. Since the best candidates in every generation satisfy the structural constraint, the maximum fitness value is the same as the maximum lift-to-drag ratio. The average fitness did not converge to the maximum fitness since some design candidates had very low fitness doe to violation of the structural constraint. The lift-to-drag ratio has increaced to 19.84 while the design obtained by sequentioal coding had the lift-to-drag ratio of 19.56.

Spanwise thickness and twist angle distributions of the optimized wing are shown in Figs. 9 and 10, respectively. The thickness distribution satisfies the structural constraint.

Figure 11 shows surface pressure contours on upper surface of the design. EA has found a design that has no flow separation and thus no pressure drag. In addition, a weak shock wave appears only at the 30% spanwise station where the maximum thickness is required. This indicates there is a tradeoff between the increase of the structural strength and the reduction of the wave drag. The resulting wing is a compromise for the given constraint.

The resulting loading distribution shown in Fig. 12 is far from the parabola that is known to give the minimum induced drag when the structural constraint is considered[5]. This indicates the design does not achieve the minimum induced drag. However, to reduce the induced drag further, the lift at the midspan region has to be increased. This will resulting a stronger shock wave and thus larger wave drag. Again there is a tradeoff between reductions of the induced drag and wave drag and the present optimizer has found a good comprimise.

4 CONCLUSION

Taguchi method has been used to analyze epistasis in the design variables of a transonic wing shape design. The coding structure for EA is modified according to the resultant epistasis information.

To examine the performance of the new coding technique, multiobjective optimizations were first performed by using an inviscid flow code, FLO-27. The present EA with the new coding technique has found the supercritical wing while the EA with the conventional coding has fallen into a premature convergence. These results indicate the validity of the present approach.

Next, a single-objective optimization using EA coupled with the three-dimensional N-S solver was demonstrated. To overcome enormous computational time necessary for this optimization, the N-S evaluations were processed in parallel on NWT. The design obtained by EA with the new coding technique had higher lift-to-drag ratio than the design obtained by EA with the sequential coding. Tradeoffs are found among increase of the structural strength, reduction of the wave drag and reduction of the induced drag. The present EA was succeeded in finding a good compromised design that have a fully attached flow and a weak shock wave only at the kink region where the structural constraint requires the maximum thickness.

ACKNOWLEDGEMENTS

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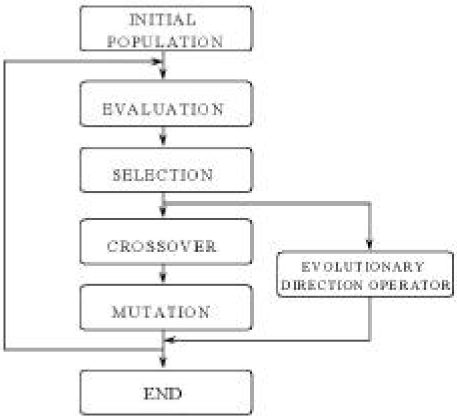

Figure 1. Flowchart of Evolutionary Algorithm

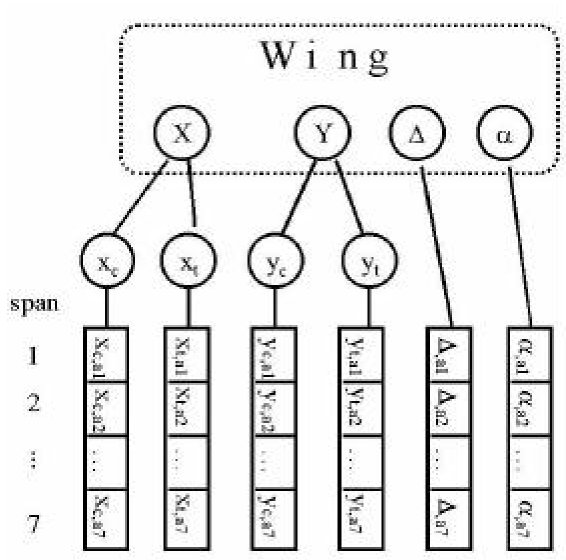

Figure 3. Tree structure of design variables

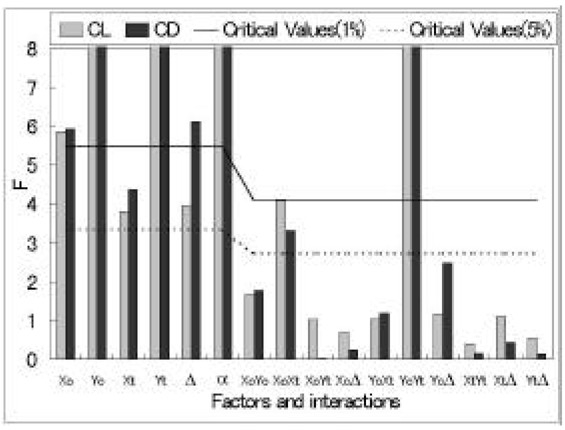

Figure 2. Effectiveness of factors and their interactions

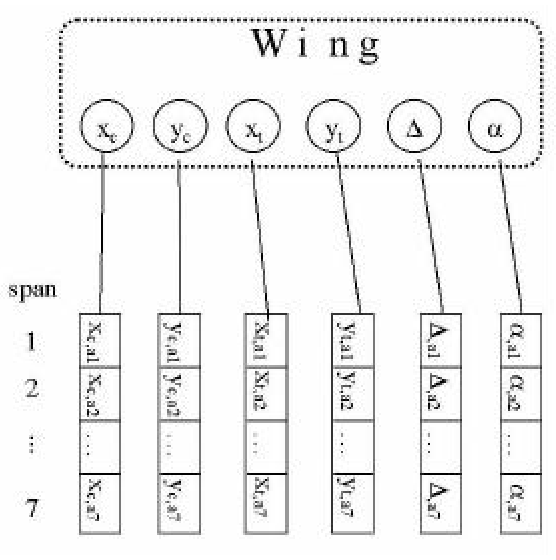

Figure 4. Sequential structure of design variables

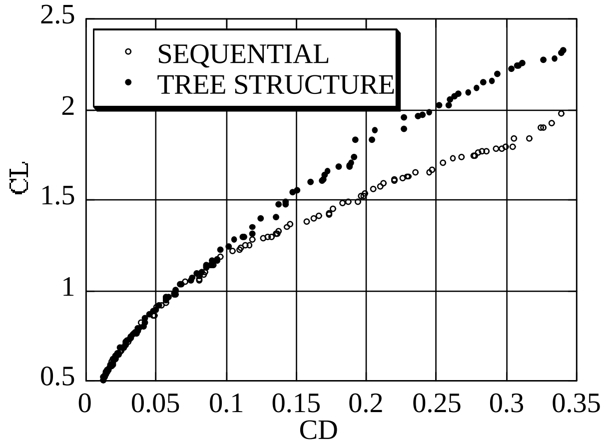

Figure 5. Pareto optimum of a wing optimization

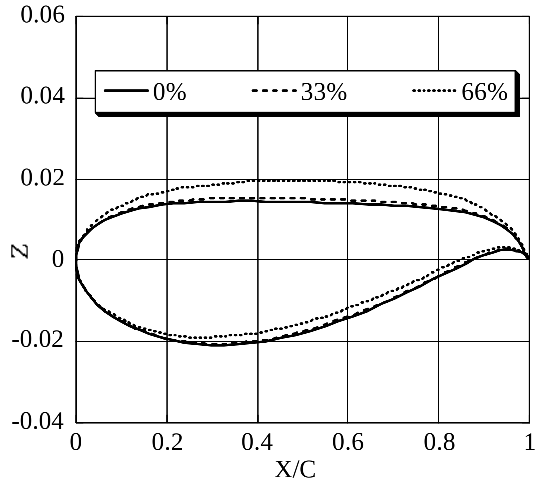

Figure 7. Wing shape optimized by tree structure coding

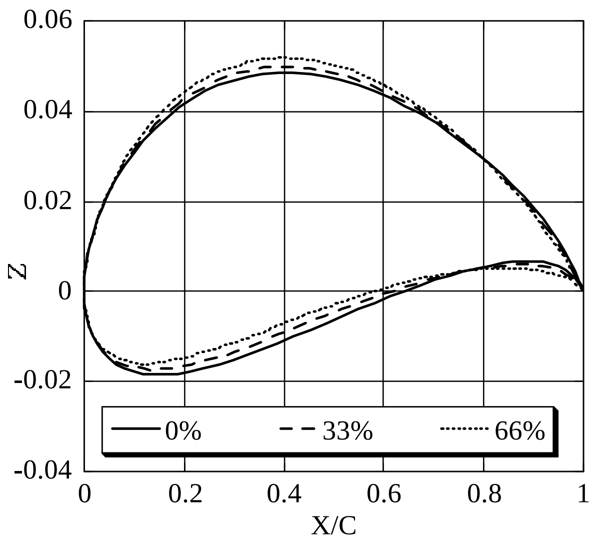

Figure 6. Wing shape optimized by sequential coding

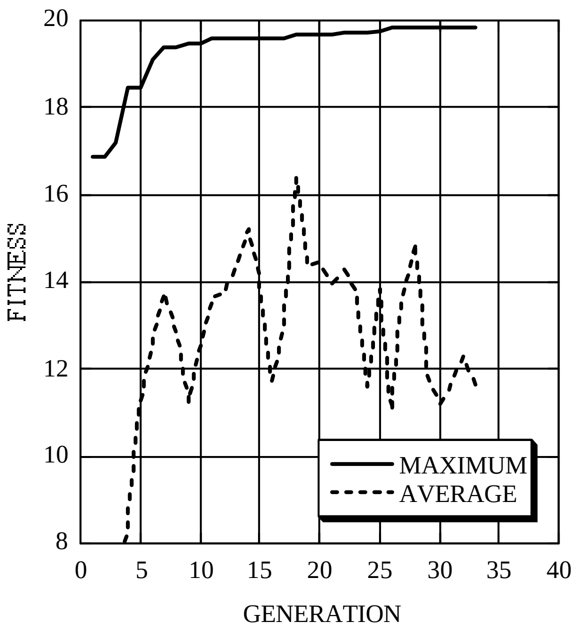

Figure 8. Optimization history

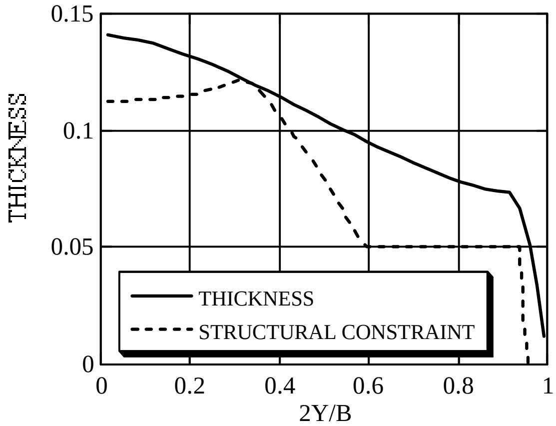

Figure 9. Spanwise thickness distribution

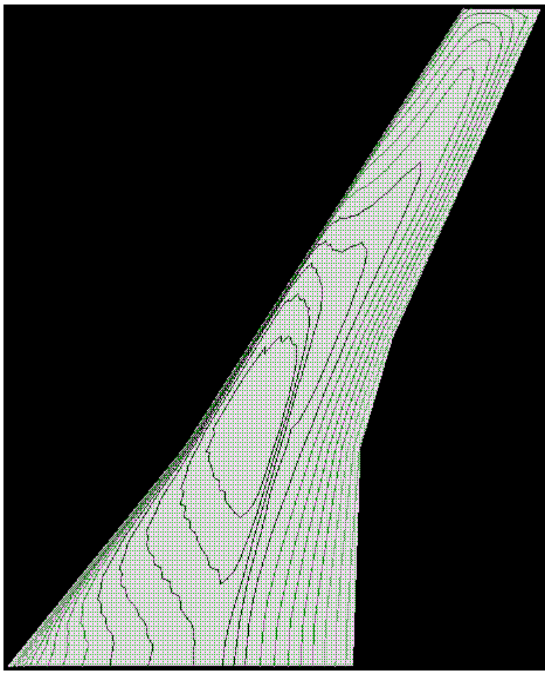

Figure 11. Pressure contours on the upper surface of the wing

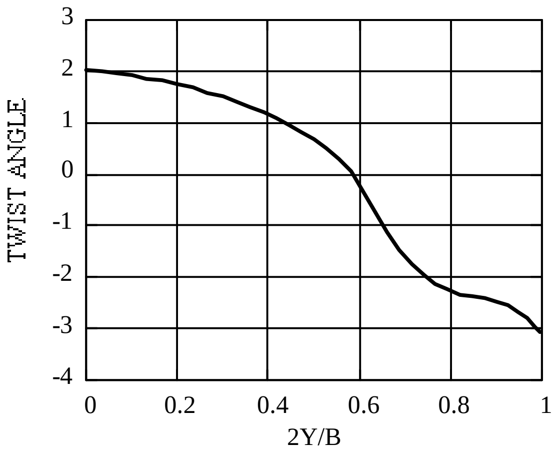

Figure 10. Spanwise twist angle distribution

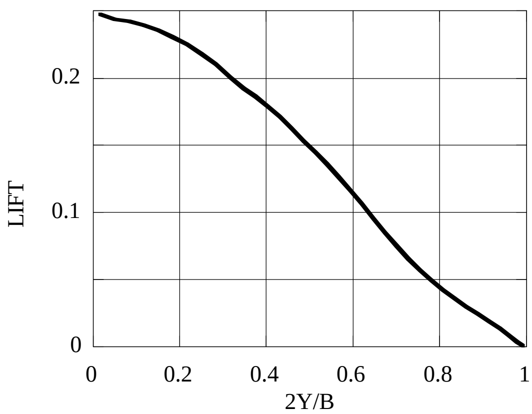

Figure 12. Spanwise load distribution