FRACTIONAL FACTORIAL DESIGN OF GENETIC CODING FOR AERODYNAMIC OPTIMIZATION

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ABSTRACT

Evolutionary Algorithms (EAs) based on structured coding have been proposed for aerodynamic optimization of wing design. Fractional factorial design is used to investigate interactions of the design variables to determine the appropriate coding structure for EAs in advance. To improve efficiency and accuracy of this approach, parameterization techniques of airfoil shapes are first tested through reproduction of a NASA supercritical airfoil. Their performance is also examined by performing aerodynamic design optimization coupled with a two-dimensional Navier-Stokes code. Finally, three-dimensional wing design is optimized based on a potential flow code and the design result is presented.

1. INTRODUCTION

In aerodynamic designs, Evolutionary Algorithms (EAs, for example, see [1]) have increasingly become popular due to the remarkable robustness and simplicity in coupling CFD codes together. EAs have been successfully applied to aerodynamic designs of two-dimensional shapes such as airfoils and turbine blades [2-4]. Even a three-dimensional wing design has been demonstrated by simplifying the geometry definition according to subsonic aerodynamics [5].

However, application of EAs to real-world problems in aerodynamic optimization may not be straightforward. Since such optimization problems usually require a large number of design variables, they have a highly multi-dimensional search space and extremely complicated objective function distribution. Without narrowing down the search region, even EAs would fail to find a globally optimum design. A transonic wing design might be a typical case.

When EAs are applied to engineering optimization problems, the complexity in the objective function appears as interactions among design parameters. These interactions are often referred as “epistasis”, corresponding to the term used in biology. If epistasis of design variables can be identified in advance, a smoother landscape of the objective function can be reproduced by rearranging the encoding of design variables. Therefore, a properly structured coding can be constructed by the epistasis analysis. However, an exhaustive search of epistasis requires as many CFD analysis's, as those required for EAs itself. Obviously, computational effort for such preprocessing is prohibitive.

In [6], EA with the structured coding have been applied to a transonic wing shape design, where wing profiles were defined by the extended Joukowski transformation [7]. In this study, Fractional Factorial Design (FFD, [8]) was applied to examine the epistasis of the design variables. FFD is a statistical tool and it has been developed to gain sufficient information from a

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structured set of coherent tests at the least expenditure of resources.

Although the structured coding improved the design performance better than the regular sequential coding in Ref. 6, capability of the extended Joukowski transformation may be in question. The extended Joukowski transformation can reproduce various kinds of airfoils with small number of design variables, still it may not be adequate for real-world applications, especially for transonic airfoil definitions.

The objective of the present study is to construct an EA-based transonic wing shape optimization method using a more precise airfoil parameterization technique. First, the accuracy of airfoil parameterization techniques will be tested through reproduction of a NASA supercritical airfoil. Then, those techniques will be compared by performing an airfoil shape optimization based on an EA coupled with a two-dimensional Navier-Stokes solver. Finally, the three-dimensional wing shape optimization will be performed by using an EA based on the structured coding derived from FFD. The present results will be compared with those obtained by the extended Joukowski transformation.

2. APPROACH

2.1 EVOLUTIONARY ALGORITHMS

Evolutionary Algorithms are emergent numerical optimization algorithms modeled on mechanism of the natural evolution, which consists of fitness evaluation of individuals, selection according to the fitness, crossover and mutation of mating pair's genes. When EAs are applied to numerical optimizations, fitness, individual and genes usually correspond to objective function value, design candidate, and design variables, respectively. The initial candidates are usually created randomly. The flowchart of a typical EA is illustrated in Fig. 1.

One of the key features of EAs is that they search from multiple points, instead of moving from a single point like gradient-based methods. In addition, they require no derivatives or gradients of the objective function. These features lead to remarkable robustness in optimization and simplicity in coupling CFD codes together. In addition, parallel efficiency may be extremely high by using a simple master-slave concept for function evaluations, if such evaluations consume most of CPU time. Aerodynamic optimization using CFD will be a typical case.

To improve the robustness of EAs further, Adaptive Range Genetic Algorithm (ARGA) [9,10] was introduced in the two-dimensional optimization. ARGA were originally developed by Arakawa for binary-coded Genetic Algorithms [10]. The ARGA can overcome difficulty in solving large-scale design optimization problems promoting the population toward promising design regions during the optimization process. It was adapted for the real-number coding used here.

Extension of EAs to multiple-objective design problems is also straightforward. By using the Pareto-ranking method and the fitness sharing [11], EAs can sample possible tradeoff solutions globally from the design space. This gives an approximation of the tradeoff surface in the design space. In this study, the best-N selection [12] is incorporated, where the best N individuals are selected for the next generation among N parents and N children so that Pareto solutions will be kept once they are formed.

2.2 FRACTIONAL FRACTORIAL DESIGN

A parametric study is often conducted by varying one parameter at a time or by trial and error for a limited number of parameters. However, such approaches only lead to incomplete knowledge for a large design space. An exhaustive search, in contrast, requires unacceptably large number of experiments and thus they are not suitable to real-world problems. For instance, a complete study of a design space of 10 parameters with 3 levels requires $3^{10} = 59049$ experiments.

FFD is a statistical approach that has been developed to gain sufficient information from a structured set of coherent tests at the least expenditure of resources. It reduces the required number of experiments by arranging the experiments according to the orthogonal array. The effectiveness of factors and their interactions are estimated by F-tests. FFD was originally developed by R.A. Fisher. These days, FFD is often used for screening experiment of the response surface method.

3. AIRFOIL PARAMETERIZATION

3.1 THE EXTENDED JOUKOWSKI TRANSFORMATION

Equation (1) is well known as the Joukowski transformation that transforms a circle into various airfoils:

$$\varsigma = z + \frac{1}{z} \quad (1)$$

The extended Joukowski transformation [7] give further variety in the resultant airfoil shapes using a preliminary transformation before the Joukowski transformation as

$$z' = z - \frac{\varepsilon}{(z-\Delta)} \quad (2)$$

$$\varsigma = z' + \frac{1}{z'} \quad (3)$$

where ε and Δ are a complex number and a real number, respectively. The airfoil shape is defined by 5 parameters: center of the circle Z_c, real part and imaginary part of ε, and Δ. An example of the extended Joukowski transformation is illustrated in Fig. 2.

Instead of the raw design variables (Z_c, ε, Δ), the present design variables are given by (x_c, y_c, x_t, y_t, Δ) where the center of the circle Z_c and the complex number ε correspond to the position (x_c, y_c) and (x_t, y_t), respectively. Δ is the preliminary movement in the real axis. It is known that x_c and x_t are related to the airfoil thickness while y_c and y_t are related to the airfoil camber.

3.2 THE INVERSE THEODORSEN TRANSFORMATION

Any given airfoil shape can be transformed into a unit circle, using the Joukowski transformation and a primary approximation as

$$\varsigma = z' + \frac{1}{z'} \quad (4)$$

$$z' = z \exp\left(\sum_{0}^{\infty} \frac{c_n}{z^n}\right) \quad (5)$$

where the complex numbers C_n are determined by the Fast Fourier Transformation [13]. The inverse Theodorsen transformation can be used to parameterize an airfoil shape. The accuracy of this technique depends on the truncation of summation in Eq. (5). In this study, an airfoil shape is defined by 13 parameters, which corresponds to the truncation at the sixth term.

3.3 B-SPLINE CURVES

Parameterization using the third-order B-Spline curves is one of the most popular approaches for airfoil designs. The design parameters are positions of control points of the B-Spline curves. In this study, airfoil geometry is split into a mean camber line and thickness distribution. Five control points are used for each of the mean camber line and the thickness distribution (Fig. 3). Since locations of the leading edge and trailing edge are frozen, 12 design variables are required to give an airfoil shape.

3.4 ORTHOGONAL SHAPE FUNCTIONS

There is a class of parameterization techniques based on linear combination of shape functions. I-Chung Chang et al. have proposed an polynomial function to parameterize upper and lower surfaces of an airfoil using orthogonal shape functions to reduce the required design parameters [14]:

$$Z = a_1(x^{\frac{1}{2}} - x) + \sum_{n=2}^{6} a_n (x^{n-1} - x^n) + \sum_{n=7}^{10} a_n (x^{\frac{1}{n-4}} - x^{\frac{1}{n-5}}) \quad (6)$$

This approach for the airfoil parameterization is a derivative of the original NACA method [15]. The number of parameters is 20.

3.5 SOBIECZKY SHAPE FUCTIONS

Another approach using linear combination of shape functions is proposed by Sobieczky [16]. The key concept is that the choice of design parameters should be based on the flow structure around an airfoil and therefore, aerodynamic performance. An airfoil shape is defined by basic geometric parameters instead of the coefficients of shape functions themselves: leading-edge radius, upper and lower crest location including curvatures, trailing-edge ordinate, thickness, direction and wedge angle. Those parameters are illustrated in Fig. 4. The following polynomials are used for the geometry definition.

$$Z = \sum_{n=1}^{6} a_n \cdot X^{n-1/2} \quad (7)$$

In this study, trailing-edge thickness is frozen to 0. In total, 10 design variables are used to give an airfoil shape.

4. RESULTS

4.1 REPRODUCTION OF NASA SUPERCRITICAL AIRFOIL

To examine accuracy of airfoil parameterization techniques, reproduction of a NASA supercritical airfoil SC(2)-0414 was first performed by means of EAs minimizing differences between the SC(2)-0414 airfoil geometry and airfoil shapes reproduced by the parameterizations. To improve the capability of finding a global optimum, a real-coded ARGA was used. In the evolutionary process, 300 generations were run for five times with a population size of 200. Here, trailing-edge ordinate of the Sobieczky airfoil was frozen to 0 for comparison purpose.

The reproduced airfoils and the corresponding residuals are presented in Fig. 5 and Table. 1, respectively. It should be noted that five runs were performed for each airfoil but the difference was negligible. The results show that the SC(2)-0414 airfoil is included in the parameterized space of Sobieczky, Theodorsen, and B-Spline airfoils. The residuals of those airfoils in Table 1 are not converged to machine-zero due to the brunt trailing edge of the SC(2)-0414 airfoil.

The extended Joukowski airfoil and the airfoil using orthogonal shape functions have failed to reach the SC(2)-0414 airfoil. Obviously, failure of the extended Joukowski airfoil is due to the insufficient design space. Although the set of the orthogonal shape functions includes the generator of the NACA 4-digit airfoils, EA using the orthogonal shape functions has also failed to reach the NACA 2412 airfoil (the result is not shown here). These results indicate that the airfoil definition using orthogonal shape functions is not suited for EA-based optimization because of the difficulty in the optimization process.

4.2 TWO-DIMENSIONAL AERODYNAMIC DESIGNS

Next, aerodynamic optimizations of an airfoil shape were demonstrated to examine the airfoil parameterizations further. The objective function was the lift-to-drag ratio to be maximized. The free stream Mach number and the angle of attack were set to 0.8 and 2 degrees, respectively. The aerodynamic performance of each design was evaluated by the two-dimensional Navier-Stokes solver based on a TVD-type upwind differencing [17], the LU-SGS scheme [18] and the multigrid method [19]. The airfoil thickness was constrained so that the maximum thickness was greater than 12% of the chord length. The real-coded ARGA was used to maximize the objective function where the population size and the number of generations were both 100. Two trials were performed for each airfoil parameterization. Trailing-edge ordinate of the Sobieczky airfoil was again frozen to 0 for comparison purpose.

Figure 6 shows the optimization history. The aerodynamic performances of the design results are summarized in Table 2. These results illustrate that the performance of the designed airfoil greatly depends on the choice of the parameterization techniques.

EA using the Sobieczky shape functions has reached to the airfoil shape of the best performance. The present good design probably originates in the selection of design parameters that are directly related to the knowledge of transonic flows around the airfoil. This parameterization gives the design space wide enough as shown in the previous section, which also helps finding a global optimum.

The resulting airfoil shape and the corresponding Cp distribution are shown in Fig. 7. The surface pressure distribution is similar to that of NASA supercritical airfoils, such as an approximately uniform distribution (rooftop) on the upper surface, a weak shock wave significantly aft of the midchord, a pressure plateau downstream of the shock wave, a relatively steep pressure recovery on the extreme rearward region, and a trailing edge pressure slightly more positive than ambient pressure [20]. The design result indicates the feasibility of the present approach.

EA using the B-Spline curves has succeeded in finding a reasonably good airfoil design but the performance of the resulting airfoil is slightly less than that optimized by the Sobieczky shape functions. The reason is probably the selection of the design parameters. The locations of B-Spline control points are not related to the flow physics in contrast to the parameter set of the Sobieczky shape functions.

The resulting airfoil obtained from the inverse Theodorsen transformation performed worst. To check whether the design obtained by the Sobieczky shape functions is included in the search space of the inverse Theodorsen transformation, airfoil reproduction was tried as shown in Fig. 8. While the extended Joukowski airfoil has failed to express the best design, the others including the inverse Theodorsen transformation have succeeded. This indicates that the worst performance of EA using the inverse Theodorsen transformation is not due to insufficient search space. The reason of the failure is again that the design parameters of the inverse Theodorsen transformation are not related directly to the flow physics about an airfoil.

The extended Joukowski transformation has found an airfoil that performs better than the inverse Theodorsen transformation. This is due to a relatively small number of design parameters and thus a smaller search space. The other reason is that (x_c x_t) and (y_c y_t) are related to the airfoil thickness and the mean camber line, respectively. They are important factors for airfoil performance. As shown in Fig. 8, however, the design space is too small to obtain a global optimum for the airfoil design.

4.3 THREE-DIMENSIONAL AERODYNAMIC DESIGNS

In this section, EAs based on the structured coding were applied to a three-dimensional wing shape design using the Sobieczky shape functions. The epistasis of the design variables was examined prior to the optimization by FFD. For the comparison, the results of the wing design using the extended Joukowski transformation [6] are presented at first. Next, the structured coding is developed for the parameter set of the Sobieczky shape functions based on FFD. The resulting EA is then applied to the same optimization problem.

Objective functions used here are C_L to be maximized and C_D to be minimized. Therefore, Multi-Objective EA is used. The cruising Mach number is assumed to be 0.8. Aerodynamic performances are evaluated by the FLO-27 code, which is a conservative full-potential code developed by Jameson and Caughey [21]. The wing shape is defined by airfoil shapes and a twist distribution α given at seven spanwise sections for the extended Joukowski transformation and at four spanwise sections for the Sobieczky airfoil. The wing planform was taken from a typical transonic aircraft as Ref. 5.

4.3.1 WING DESIGN USING THE EXTENDED JOUKOWSKI TRANSFORMATION

In Ref. 6, FFD was applied to the epistasis analysis, that is, the interactions of the design variables for the extended Joukowski transformation. Analysis of interactions of all design variables for the wing model, however, requires unacceptably large number of CFD runs even with the FFD. Therefore, the design variables are grouped into the spanwise variations of airfoil shape parameters and twist angle x_c, y_c, x_t, y_t, Δ and α as shown in Fig. 9. Factors examined are the parameters themselves and their two-factor interactions except for those related to α. Three types of spanwise variations are considered as levels: no variation, linear increase from root to tip, and vice versa.

Examined responses are C_L and C_D of the wing. Only to account for positive responses in aerodynamic performance (increase in C_L and decrease in C_D), following two functions are introduced:

$$F1 = \max (C_L - C_{L0} , 0) \quad (7)$$

$$F2 = - \min (C_D - C_{D0} , 0) \quad (8)$$

where C_{L0} and C_{D0} are those of a wing having a constant airfoil section along the spanwise direction.

Figure 10 shows the result of the epistasis analysis. The solid and broken lines are critical F values with 1% and 5% statistical risks, respectively. F values more than these critical values are judged effective. While every factor is effective on both F1 and F2, interactions of $x_c x_t$ and $y_c y_t$ appear effective. This result is consistent with the fact that x_c, x_t, are related to the airfoil thickness while y_c and y_t are related to the airfoil camber line.

To make use of identified hierarchy of the design variables, spanwise distributions of airfoil parameters and twist angle are coded as genes and one-point crossover is used where the crossover of (x_c, x_t) and (y_c, y_t) are apt to happen at the same gene sites. The resultant structured coding is shown in Fig. 11. This is supposed to preserve good building blocks of (x_c, x_t) and (y_c, y_t).

EA using this coding technique was compared with EA using the regular sequential coding based on Fig. 9. Figure 12 shows the Pareto optimal solutions indicating the tradeoff between maximization of C_L and minimization of C_D. Solid and hollow points show the resulting Pareto fronts obtained from the structured and sequential codings, respectively. This figure confirms that the present EA with the structured coding outperforms the conventional EA with the sequential coding. Airfoil sections of the designed wings at C_L = 0.5 are shown in Figs. 13 and 14. The structured coding results in the shape closer to supercritical airfoils than the sequential coding.

4.3.2 WING DESIGN USING THE SOBIECZKY SHAPE FUNCTIONS

FFD is now applied to the epistasis analysis of the parameter sets for the Sobieczky shape functions. The factors to be examined are r_{LE}, X_{UP}, Z_{UP}, Z_{XXUP}, X_{LO}, Z_{LO}, Z_{XXLO}, α_{TE}, Z_{TE} and their two-factor interactions on F1 and F2. β_{TE} and its interactions are neglected since the wedge angle at the trailing edge is primary determined by the structural strength. Also, interactions of $r_{LE}Z_{TE}$, $r_{LE}\alpha_{TE}$ and $r_{LE}Z_{XXLO}$ are disregarded. Consequently, 42 factors are examined. Number of CFD runs required for this epistasis analysis is reduced from $3^9 = 19683$ to $3^6 = 729$ thanks to FFD.

Figure 15 shows the result of the epistasis analysis. Interactions effective in both C_L and C_D are illustrated with bold lines in Fig. 16. Since these figures indicate complicated interactions among the design variables, especially, Z_{UP}, Z_{LO}, and Z_{TE}, it is difficult to construct a structured coding for these design variables. Therefore, new parameters Z_C and Z_H are introduced instead of Z_{UP} and Z_{LO} as;

$$Z_C = (Z_{UP} + Z_{LO}) / 2 \qquad (9)$$

$$Z_H = (Z_{UP} - Z_{LO}) \qquad (10)$$

where Z_C and Z_H correspond to airfoil camber and thickness, respectively. Using these parameters, interactions are greatly simplified as shown in Fig.17. According to this result, a structured coding for the spanwise distributions of airfoil parameters is introduced as shown in Fig.18. Here, one-point crossover is apt to happen at the same gene sites for the connected spanwise distributions in the figure.

The design result obtained by the EA using this structured coding is again compared with that obtained by the EA using the sequential coding. Figure 19 shows Pareto fronts obtained from the sequential coding of the original Sobieczky shape functions and the structured coding using Z_C and Z_H. The advantage of the structured coding is clearly observed. The corresponding designs are shown in Figs. 20 and 21. The drag coefficients at $C_L = 0.5$ are summarized in Table 3. The best design is obtained by EA based on the proposed structured coding using the Sobieczky shape functions.

5. CONCLUSIONS

First, several parameterization techniques for the airfoil definition have been investigated through the reproduction of a NASA supercritical airfoil and the aerodynamic optimization by means of EAs. The results show that the performance of the designed airfoil greatly depends on the choice of the parameterization techniques. EA using the Sobieczky shape functions succeeded in finding the airfoil shape of the best performance. This is thanks to the selection of design parameters that are directly related to the knowledge of transonic flows around the airfoil. This parameterization gives the design space wide enough, which also helps finding a global optimum.

Next, FFD was applied to the epistasis analysis of the parameter set for the Sobieczky shape functions. FFD was used to analyze interactions of the design variables. The coding structure for EA was developed according to the resultant information. EA using the structured coding was then applied to the aerodynamic optimization problem of a transonic wing design. The design results indicate that the structured coding for EAs is a promising approach to find a global optimum in real-world applications.

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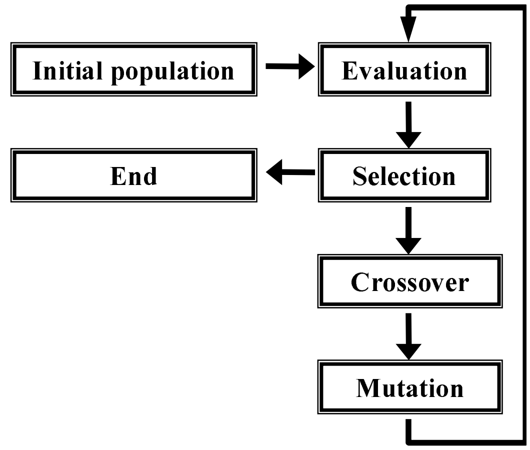

Figure 1. Flowchart of Evolutionary Algorithms.

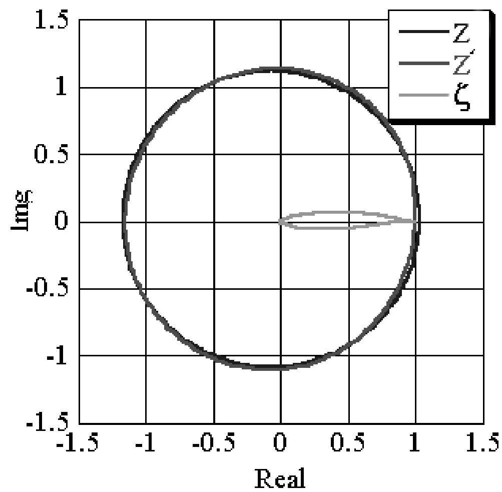

Figure 2. Example of the extended Joukowski transformation.

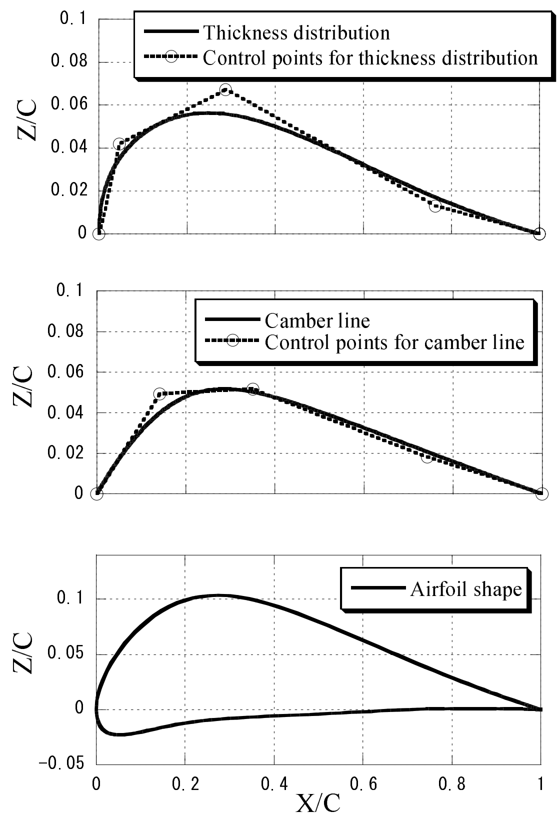

Figure 3. B-Spline curves for mean camber line and thickness distribution and the resultant airfoil shape.

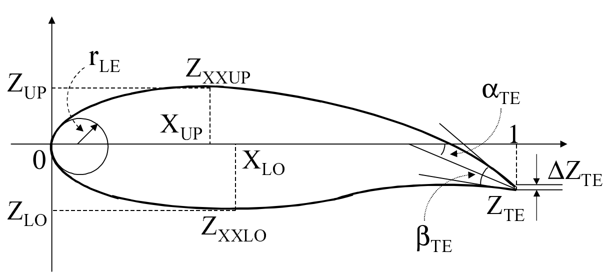

Figure 4. Design parameters for the Sobieczky shape functions.

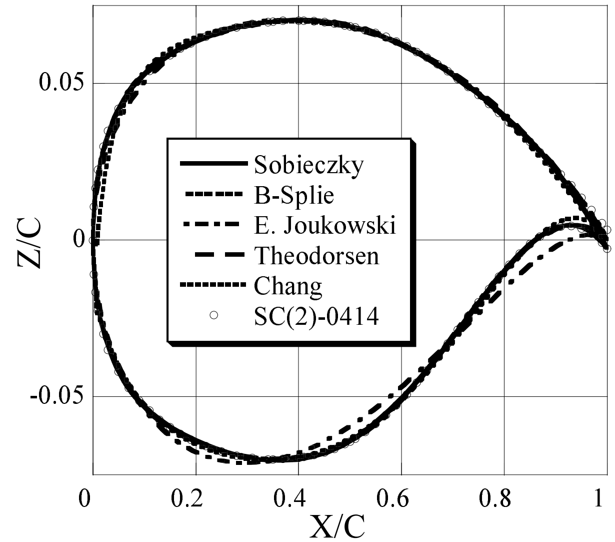

Figure 5. Comparison of the reproduced airfoils.

Table 1. Residual of the SC(2)-0414 airfoil reproduction.

Extended Joukowski	Chang	Sobieczky	Theodorsen	B-Spline
4.17e-3	2.40e-3	8.00e-4	7.02e-4	7.13e-4

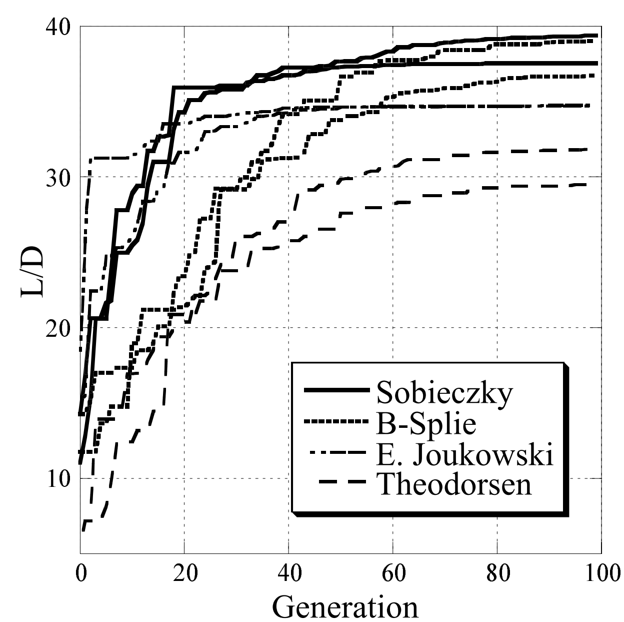

Figure 6. Optimization histories.

Table 2. Results of aerodynamic optimizations.

	Theodorsen	Extended Joukowski	B-Spline	Sobieczky
L/D	31.87	34.73	39.02	39.40
Cl	0.5535	0.5427	0.6223	0.6253
Cd	0.01737	0.01562	0.01595	0.01587

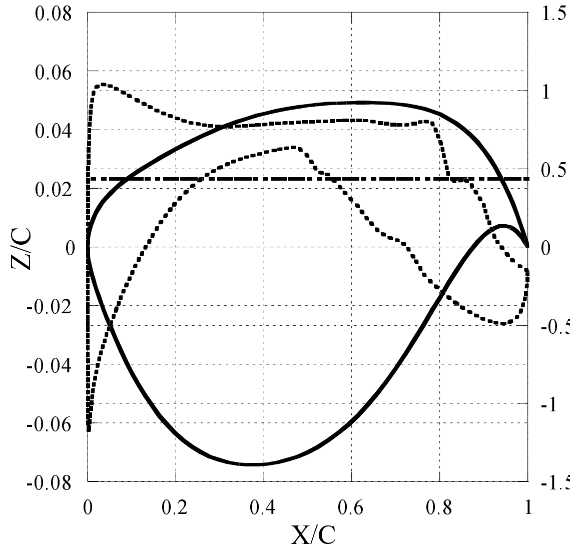

Figure 7. Designed airfoil shape and the corresponding pressure distribution using Sobieczky shape functions.

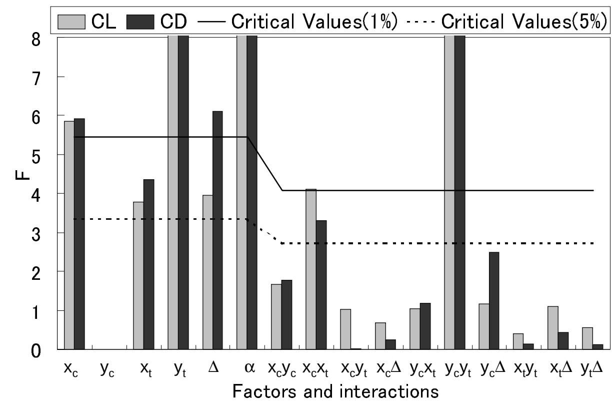

Figure 10. Effectiveness of factors and their interactions for the extended Joukowski transformation.

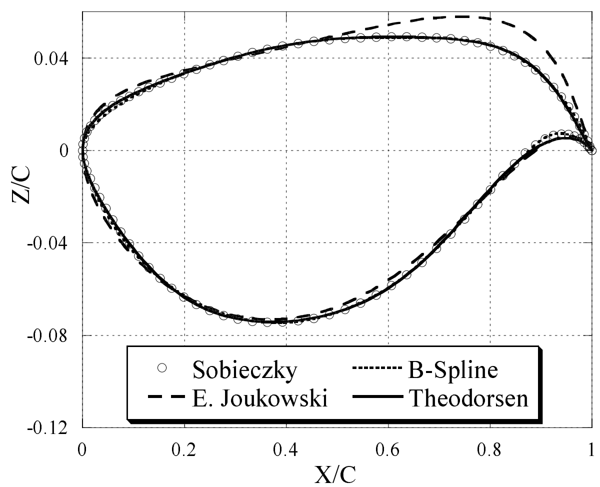

Figure 8. Reproduction of the airfoil design obtained by Sobieczky shape functions.

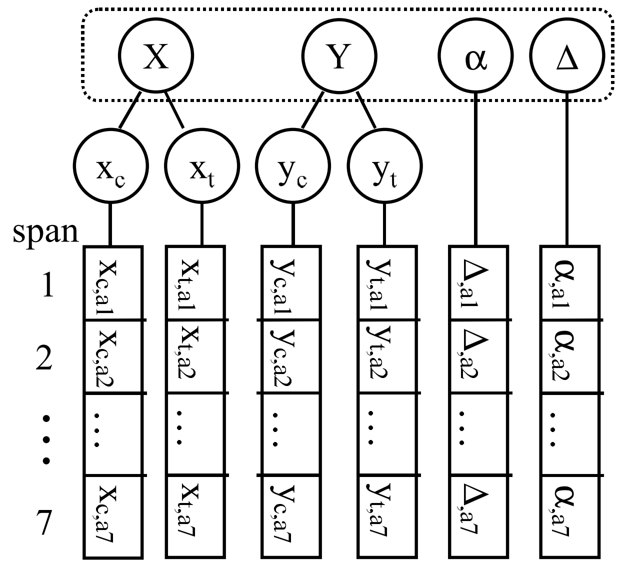

Figure 11. Structured coding for the extended Joukowski transformation.

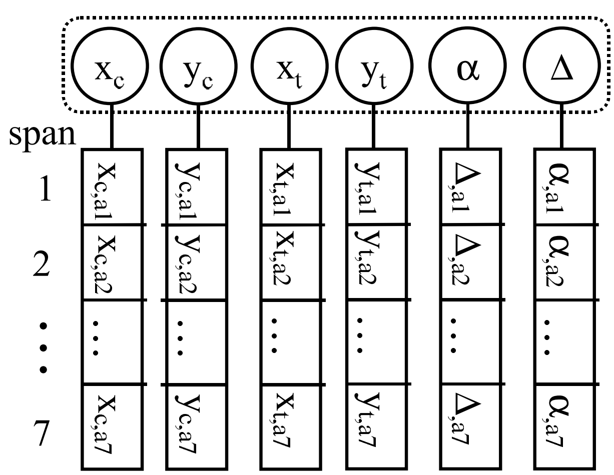

Figure 9. Sequential coding for the extended Joukowski transformation.

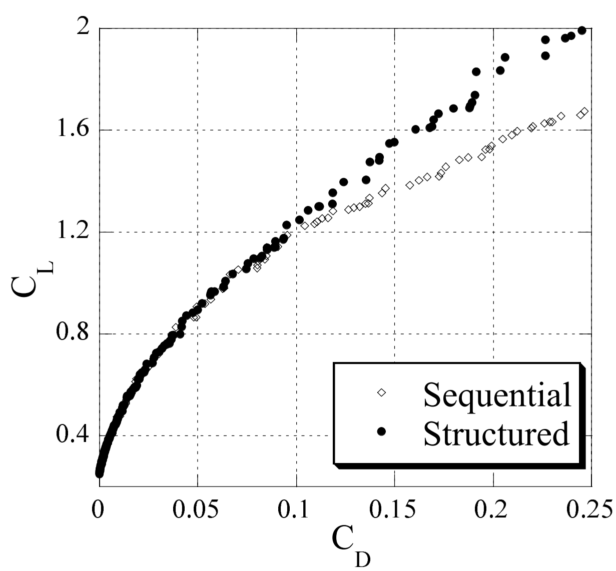

Figure 12. Comparison of Pareto fronts for sequential and structured coding techniques.

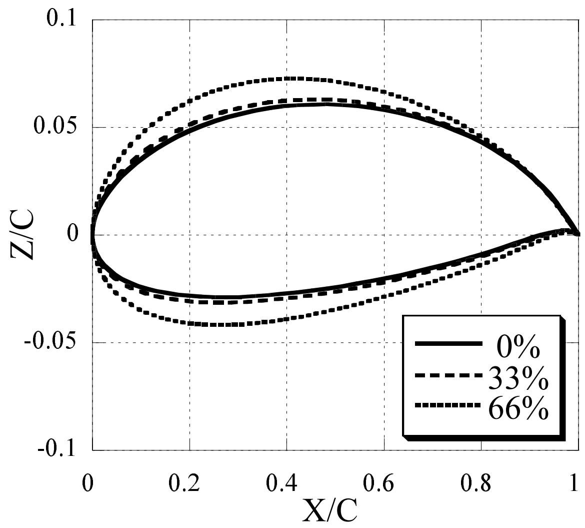

Figure 13. Wing shape optimized by sequential coding.

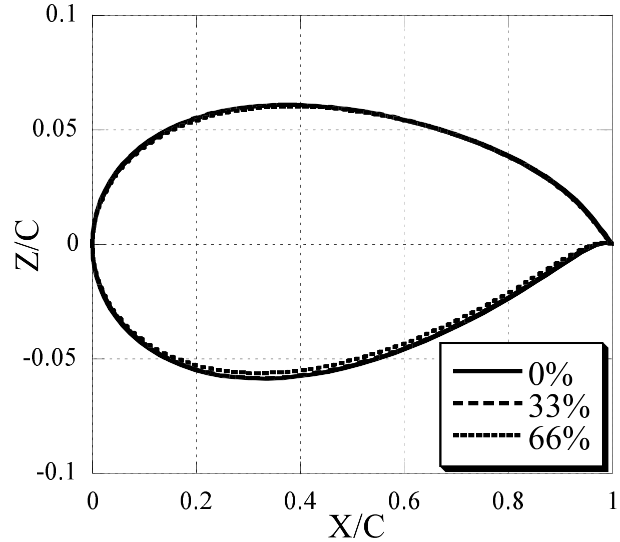

Figure 14. Wing shape optimized by structured coding.

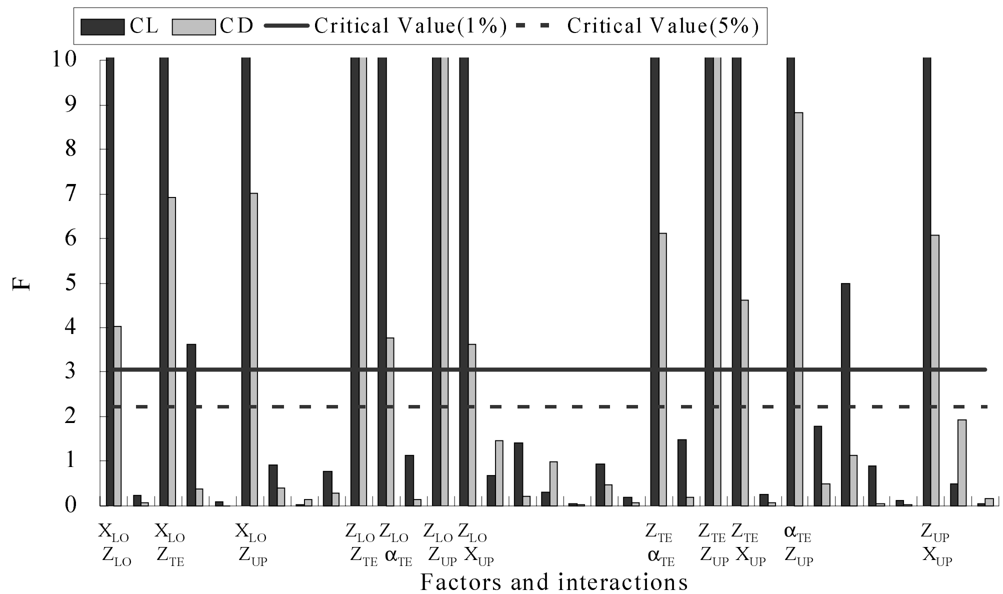

Figure 15. Effectiveness of factors and their interactions for Sobieczky shape functions.

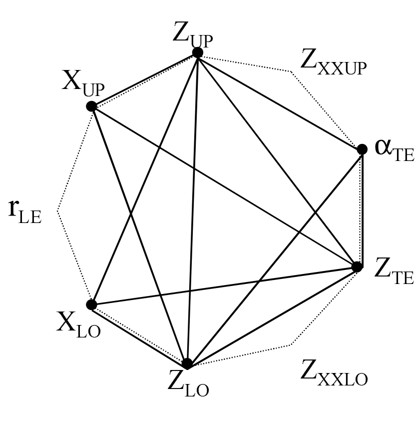

Figure 16. Effective interactions of the original airfoil definition.

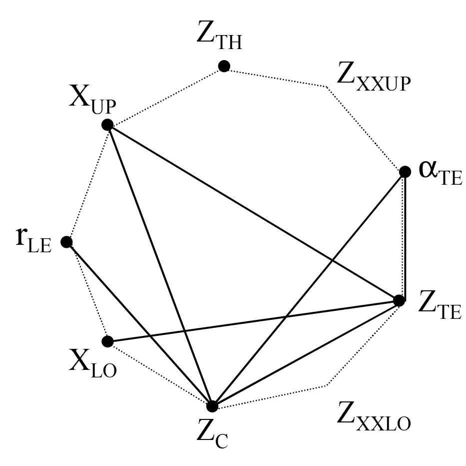

Figure 17. Effective interactions of the modified airfoil definition.

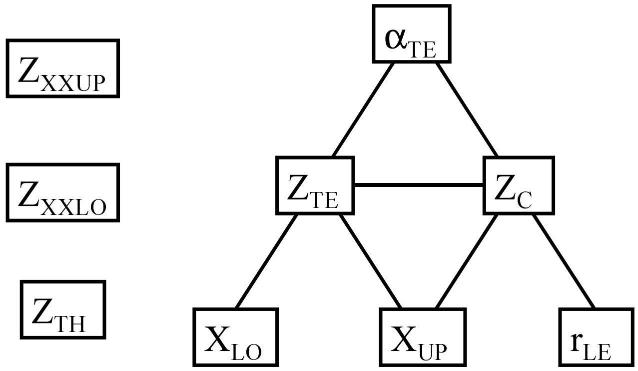

Figure 18. Structured coding for the Sobieczky airfoil.

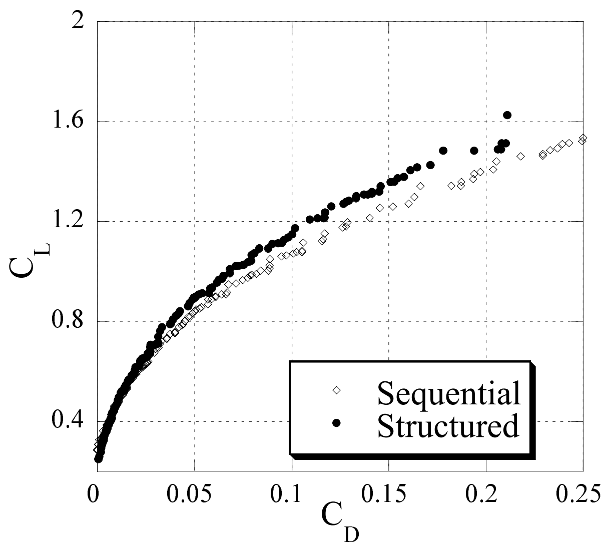

Figure 19. Comparison of Pareto fronts for sequential and structured coding techniques.

Table 3. Comparison of C_D at C_L=0.5.

	Sequential coding	Structured coding
Extended Joukowski transformation airfoil	1.1961e-2	1.1704e-2
Sobieczky airfoil	1.1951e-2	1.1339e-2

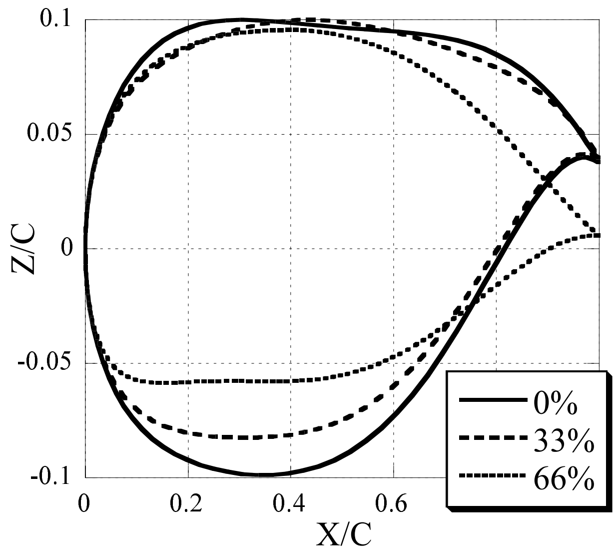

Figure 20. Wing shape optimized by sequential coding.

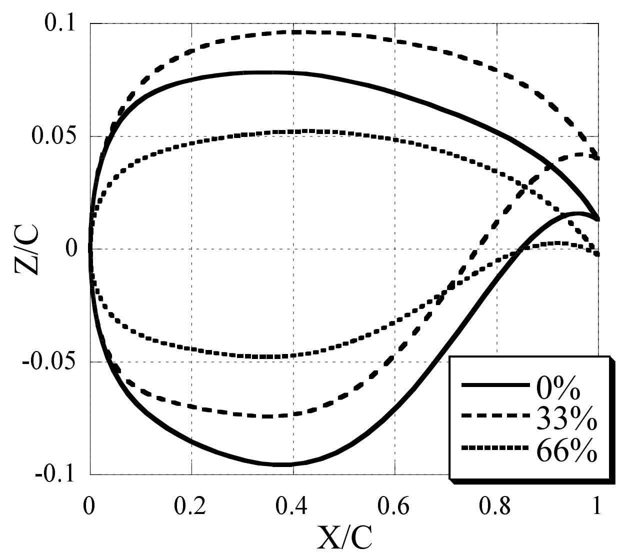

Figure 21. Wing shape optimized by structured coding.